Photoproduction and Radiative Decay of Spin 1/2 and 3/2 Pentaquarks

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Abstract

We study photoproduction and radiative decays of pentauqarks paying particular attention to the differences between spin-1/2 and spin-3/2, positive and negative parities of pentaquarks. Detailed study of these processes can not only give crucial information about the spin, but also the parity of pentaquarks.

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I. INTRODUCTION

Recently several experiments have reported evidences for pentaquarks Θ and other states [1, 2, 3]. The first observed pentaquark state was the $\Theta(1540)$ with strangeness S = +1and was identified as a state with quark content $udud\bar{s}$. This particle is an isosinglet and belongs to the anti-decuplet multiplet in flavor $SU(3)_f$ symmetry [4]. Consequently NA49 has reported evidences for isoquated $\Xi_{3/2}$ in the anit-decuplet [2]. At present there are very limited information on the detailed properties such as the spin, the parity and the magnetic dipole moment. Several other experiments have also carried out searches for these particles. Some of them reported positive and while others reported negative results [3]. One has to wait future experiments to decide whether these pentaquark state are real. On the theoretical front, there are also many studies trying to understand the properties of these possible pentaquark states [5, 6, 7, 8, 9]

In this paper we explore possibilities of studying the properties of pentaquark Θ and its partners in the SU(3) anti-decuplet multiplet, using radiative processes involving a pentaquark P, an ordinary baryon N and a pseudoscalar Π . We consider two classes of processes, the photoproduction $\gamma + N \to \Pi P$ and radiative decay $P \to N \Pi \gamma$.

In the above N and Π indicate a member in the ordinary baryon octet and pseudoscalar octet of $SU(3)_f$, respectively. They are given by

$$N = (N_i^j) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \ \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \ n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad \Pi = (\Pi_i^j) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \ \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \ K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} (1)$$

P is a member of the anti-decuplet ($\overline{10}$) pentaquark multiplet. This multiplet has 10 members which can be described by a totally symmetric tensor P^{ijk} in SU(3). The 10 members are

$$P^{111} = \Xi_{3/2}^{--}, \quad P^{112} = \Xi_{3/2}^{-} / \sqrt{3}, \quad P^{122} = \Xi_{3/2}^{0} / \sqrt{3}, \quad P^{222} = \Xi_{3/2}^{+},$$

$$P^{113} = \Sigma_{a}^{-} / \sqrt{3}, \quad P^{123} = \Sigma_{a}^{0} / \sqrt{6}, \quad P^{223} = \Sigma_{a}^{+} / \sqrt{3},$$

$$P^{133} = N_{a}^{0} / \sqrt{3}, \quad P^{233} = N_{a}^{+} / \sqrt{3}, \quad P^{333} = \Theta^{+}.$$
(2)

Without $SU(3)_f$ symmetry breaking members in a $SU(3)_f$ multiplet all have the same

mass. The degeneracy of mass is lifted by the light quark mass differences, m_u , m_d and m_s . Using information on the masses of Θ and $\Xi_{3/2}$ including the leading $SU(3)_f$ breaking effects, the masses of the anti-decuplet members are given by[5] $m_{\Theta} = 1542$ MeV, $m_{\Xi_{3/2}} = 1862$ MeV, $m_{\Sigma_a} = 1755$ MeV, and $m_{N_a} = 1648$ MeV.

Discussions for radiative processes involving a P, a N, a Π and a γ with spin-1/2 pentaquarks have been carried out in several papers[5, 6]. There are also some studies for spin-3/2 pentaquarks[7], but no detailed studies of radiative processes. In this work we will consider both spin-1/2 and spin-3/2 cases and paying particular attention for the differences. Since in the processes considered involve pseudoscalar goldstone bosons π and K, we will use chiral perturbation theory to carry out the analysis.

II. THE MATRIX ELEMENTS FOR RADIATIVE PROCESSES

The leading order diagrams for the radiative processes involving a P, a N, a Π and a γ are shown in Figure 1. The electromagnetic coupling of photon with Π and N are known. To evaluate these diagrams, we need to know the various couplings involving pentaquarks.

A. The spin-1/2 case

There are two types of electromagnetic couplings, the electric charge and magnetic dipole interactions. The leading chiral electric charge and magnetic dipole couplings are given by

$$L_{e} = \bar{P}i\gamma^{\mu}D_{\mu}P = \bar{P}_{ijk}i\gamma^{\mu}(\partial_{\mu}P^{ijk} - V^{i}_{\mu,l}P^{ljk} - V^{j}_{\mu,l}P^{ilk} - V^{k}_{\mu,l}P^{ijl}),$$

$$L_{m} = \frac{\mu_{P}}{4}\bar{P}_{ijk}\sigma^{\mu\nu}(f^{i}_{\mu\nu,l}P^{ljk} + f^{j}_{\mu\nu,l}P^{ilk} + f^{k}_{\mu\nu,l}P^{ijl}),$$
(3)

where $V_{\mu} = (1/2)(\xi^{\dagger}\partial_{\mu}\xi + \xi\partial_{\mu}\xi^{\dagger}) + i(e/2)A_{\mu}(\xi^{\dagger}Q\xi + \xi Q\xi^{\dagger})$. Here $\xi = exp[i\Pi/\sqrt{2}f_{\pi}]$ and Q = Diag(2/3, -1/3, -1/3) is the quark charge matrix and A_{μ} is the photon field. $f_{\mu\nu,i}^{j} = F_{\mu\nu}(\xi^{\dagger}Q\xi + \xi Q\xi^{\dagger})_{i}^{j}$ with $F_{\mu\nu}$ being the photon field strength. Expanding to the leading order, we have for each individual pentaquark

$$L_e = -eQ_i \bar{P}_i \gamma^{\mu} A_{\mu} P_i,$$

$$L_m = -\frac{e\mu_P Q_i}{2} \bar{P}_i \sigma^{\mu\nu} F_{\mu\nu} P_i.$$
(4)

We note that for neutral pentuaquarks, to the leading order the anomalous dipole moments are zero. The kappa parameter $\kappa_P = 2m_P\mu_P$ have been estimated to be of order one[8]. In our analysis we will treat it as a free parameter to see if experimental data can provide some information.

We also need to know the strong interaction coupling of a pentaquark with an ordinary baryon and a pseudoscalar. It can be parameterized as

$$L_{PN\Pi} = g_{PN\Pi} \bar{P}_{ilm} \Gamma_P \gamma^{\mu} (\tilde{A}_{\mu})^l_j N^m_k \epsilon^{ijk} + H.C.$$
(5)

In the above Γ_p takes "+1" and " γ_5 " if P has negative and positive parities, respectively. $\tilde{A}_{\mu} = (i/2)(\xi^{\dagger}\partial_{\mu}\xi - \xi\partial_{\mu}\xi^{\dagger}) - (e/2)A_{\mu}(\xi^{\dagger}Q\xi - \xi Q\xi^{\dagger}).$ Expanding the above effective Lagrangian to the leading order we obtain $P - N - \Pi$ type

Expanding the above effective Lagrangian to the leading order we obtain $P - N - \Pi$ type of couplings. The results are given in Table 1.





Θ^+	$-nK^+ + pK^0$
N_a^0	$\frac{1}{6}(-3\sqrt{2}n\eta + 3\sqrt{2}\Lambda K^0 + \sqrt{6}\Sigma^0 K^0 - \sqrt{6}n\pi^0 + 2\sqrt{3}p\pi^ 2\sqrt{3}\Sigma_a^- K^+)$
N_a^+	$\frac{1}{6}(3\sqrt{2}p\eta - 3\sqrt{2}\Lambda K^{+} + \sqrt{6}\Sigma^{0}K^{+} - \sqrt{6}p\pi^{0} - 2\sqrt{3}n\pi^{+} + 2\sqrt{3}\Sigma^{+}K^{0})$
Σ_a^-	$\frac{1}{6}(2\sqrt{3}nK^{-} + 3\sqrt{2}\Lambda\pi^{-} + \sqrt{6}\Sigma^{0}\pi^{-} - 3\sqrt{2}\Sigma^{-}\eta - \sqrt{6}\Sigma^{-}\pi^{0} - 2\sqrt{3}\Xi^{-}K^{0})$
Σ_a^0	$\frac{1}{6}(\sqrt{6}n\bar{K}^0 - \sqrt{6}pK^ 3\sqrt{2}\Lambda\pi^0 + 3\sqrt{2}\Sigma^0\eta - \sqrt{6}\Sigma^-\pi^+ + \sqrt{6}\Sigma^+\pi^ \sqrt{6}\Xi^0K^0 + \sqrt{6}\Xi^-K^+)$
Σ_a^+	$\frac{1}{6}(-2\sqrt{3}p\bar{K}^0 - 3\sqrt{2}\Lambda\pi^+ + \sqrt{6}\Sigma^0\pi^+ + 3\sqrt{2}\Sigma^+\eta - \sqrt{6}\Sigma^+\pi^0 + 2\sqrt{3}\Xi^0K^+)$
$\Xi_{3/2}^{}$	$\Sigma^- K^ \Xi^- \pi^-$
$\Xi^{3/2}$	$\frac{1}{6}(-2\sqrt{6}\Sigma^0 K^- + 2\sqrt{3}\Sigma^- \bar{K}^0 - 2\sqrt{3}\Xi^0 \pi^- + 2\sqrt{6}\Xi^- \pi^0)$
$\Xi_{3/2}^{0}$	$\frac{1}{6}(-2\sqrt{6}\Sigma^0\bar{K}^0 - 2\sqrt{3}\Sigma^+K^- + 2\sqrt{3}\Xi^0\pi^0 + 2\sqrt{6}\Xi^-\pi^+)$
$\Xi_{3/2}^+$	$-\Sigma^+ \bar{K}^0 + \Xi^0 \pi^+$

TABLE I: P-N- Π couplings in unit $g_{PN\Pi}/\sqrt{2}f_{\pi}$. The couplings in the tables are understood to be in the form $-a_{PN\Pi}\bar{P}\Gamma_P\gamma^{\mu}N\partial_{\mu}\Pi$. The coefficient in front of $N\Pi$ in the second column is $-a_{PN\Pi}$.

The contact γ -P-N- Π coupling in Figure 1.d is obtained from a term $ieg_{PN\Pi}A_{\mu}\bar{P}_{ilm}\Gamma_{P}\gamma^{\mu}[\Pi,Q]_{j}^{l}N_{k}^{m}\epsilon^{ijk}$ obtained by expanding $L_{PN\Pi}$.

In the following we display the matrix element for $P \to N\Pi\gamma$. The matrix element for $\gamma N \to P\Pi$ can be obtained by making appropriate changes of signs for the relevant particle momenta. We have

$$M(P \to N\Pi\gamma) = \frac{eg_{PN\Pi}}{\sqrt{2}f} a_{PN\Pi} \epsilon_{\mu}^* \bar{N} [Q_{\Pi} \Gamma_P \gamma^{\mu} - (Q_N \gamma^{\mu} + \frac{\mu_N}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) \frac{1}{\gamma \cdot P_{\gamma} + \gamma \cdot P_N - m_N} \Gamma_P \gamma \cdot P_{\pi} - \Gamma_P \gamma \cdot P_{\Pi} \frac{1}{\gamma \cdot P_N + \gamma \cdot P_{\Pi} - m_P} (Q_P \gamma^{\mu} + \frac{\mu_P}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) - Q_{\Pi} \frac{(2P_{\Pi} + P_{\gamma})^{\mu}}{(P_{\Pi} + P_{\gamma})^2 - m_{\Pi}^2} \Gamma_P (\gamma \cdot P_{\Pi} + \gamma \cdot P_{\gamma})] P.$$
(6)

For $\Theta^+ \to nK^+\gamma$, $a_{PN\Pi} = a_{\Theta nK} = 1$, $Q_P = Q_\Theta = 1$, $Q_N = Q_n = 0$, $Q_{\Pi} = Q_{K^+} = 1$. For $\Theta^+ \to pK^0\gamma$, $a_{PN\Pi} = a_{\Theta pK} = -1$, $Q_N = Q_p = 1$ and $Q_{K^0} = 0$. For $\Xi_{3/2}^{--} \to \Sigma^- K^-\gamma$, $a_{PN\Pi} = a_{\Xi_{3/2}^{--}\Sigma^-K^-} = -1$, $Q_P = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Sigma^-} = -1$, $Q_{\Pi} = Q_{K^-} = -1$. And for $\Xi_{3/2}^{--} \to \Xi^-\pi^-\gamma$, $a_{PN\Pi} = a_{\Xi_{3/2}^{--}\Xi^-\pi^-} = 1$, $Q_P = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -1$, $Q_{\Pi} = Q_{\pi^-} = -1$.

The parameter $g_{PN\Pi}$ can be determined from a pentaquark P decays into a baryon and

a meson. For example

$$\frac{g_{PN\Pi}^2}{2f_{\pi}^2} = \frac{\Gamma(\Theta^+ \to nK^+) 16\pi m_{\Theta}}{(m_n + \hat{P}m_{\Theta})^2 ((m_n - \hat{P}m_{\Theta})^2 - m_K^2) Phase},$$

$$Phase = \sqrt{(1 - (m_K + m_n)^2/m_{\Theta}^2)((1 - (m_K - m_n)^2/m_{\Theta}^2))}.$$
(7)

In the above " \hat{P} " is the eigenvalue of the parity, it takes "+" for positive parity and "-" for negative parity pentaquark, respectively.

From Table 1 we see that Θ^+ only has two strong decay channels, pK^0 and nK^+ . The total width of Θ^+ is therefore $\Gamma_{\Theta} = \Gamma(\Theta^+ \to pK^0) + \Gamma(\Theta^+ \to nK^+)$. If the Γ_{Θ} is determined, one can determine $g_{PN\pi}^2$ from eq.7

B. The spin-3/2 case

In this case one needs to use the Rarita-Schwinger field for pentaquarks P_{ilm}^{μ} . The electromagnetic couplings needed are modified compared with spin-1/2 particles, and they are given by

$$L_{e} = \bar{P}^{\alpha} i \gamma^{\mu} D_{\mu} P_{\alpha} = \bar{P}^{\alpha}_{ijk} i \gamma^{\mu} (\partial_{\mu} P^{ijk}_{\alpha} - V^{i}_{\mu,l} P^{ljk}_{\alpha} - V^{j}_{\mu,l} P^{ilk}_{\alpha} - V^{k}_{\mu,l} P^{ijl}_{\alpha}),$$

$$L_{m} = \frac{\mu_{P}}{4} \bar{P}^{\alpha}_{ijk} \sigma^{\mu\nu} (f^{i}_{\mu\nu,l} P^{ljk}_{\alpha} + f^{j}_{\mu\nu,l} P^{ilk}_{\alpha} + f^{k}_{\mu\nu,l} P^{ijl}_{\alpha}).$$
(8)

Since a spin-3/2 particle can have dipole and quadrupole moments, if both are not zero, one should add another term to the electromagnetic couplings,

$$L_q = \tau_P \bar{P}_\nu F^{\mu\nu} P_\mu, \tag{9}$$

We will take it to be zero in our later discussions.

The chiral Lagrangian for strong coupling involving a pentaquark, a baryon and a pseudoscalar is given by

$$L_{PN\Pi} = g_{PN\Pi} \bar{P}^{\mu}_{ilm} \gamma_5 \Gamma_P (A_{\mu})^l_j N^m_k \epsilon^{ijk} + H.C.$$
(10)

From the above we have

$$\Gamma(P \to N\Pi) = \frac{g_{PN\Pi}^2}{2f^2} \frac{Phase}{16\pi m_P} \frac{1}{3} ((\hat{P}m_P + m_N)^2 - m_{\Pi}^2) \\ \times (\frac{1}{4m_P^2} (m_P^2 + m_{\Pi}^2 - m_N^2)^2 - m_{\Pi}^2).$$
(11)

Combining the above information we obtain the matrix element for $P \to N \Pi \gamma$

$$M(P \to N\Pi\gamma) = \frac{eg_{PN\Pi}}{\sqrt{2}f} a_{PN\Pi} \epsilon_{\mu}^{*} \bar{N} [Q_{\Pi}\gamma_{5}\Gamma_{P}g^{\mu\nu} - (Q_{N}\gamma^{\mu} + \frac{\mu_{N}}{2}[\gamma^{\mu}, \gamma \cdot P_{\gamma}]) \frac{1}{\gamma \cdot P_{\gamma} + \gamma \cdot P_{N} - m_{N}} \gamma_{5}\Gamma_{P}P_{\pi}^{\nu} + \gamma_{5}\Gamma_{P}P_{\Pi}^{\alpha}G_{\alpha}^{\ \nu}(Q_{P}\gamma^{\mu} + \frac{\mu_{P}}{2}[\gamma^{\mu}, \gamma \cdot P_{\gamma}]) - Q_{\Pi}\frac{(2P_{\Pi} + P_{\gamma})^{\mu}}{(P_{\Pi} + P_{\gamma})^{2} - m_{\Pi}^{2}} \gamma_{5}\Gamma_{P}(P_{\Pi} + P_{\gamma})^{\nu}]P_{\nu}.$$
(12)

In the above $G^{\mu\nu}$ is the spin-3/2 propagator resulting from the following most general Lagrangian[10]

$$L = \bar{P}_{\mu}\Lambda^{\mu\nu}P_{\nu},$$

$$\Lambda^{\mu\nu} = (\gamma \cdot P_{P} - m_{P})g^{\mu\nu} + A(\gamma^{\mu}P_{P}^{\nu} + P_{P}^{\mu}\gamma^{\nu})$$

$$+ \frac{1}{2}(3A^{2} + 2A + 1)\gamma^{\mu}\gamma \cdot P_{P}\gamma^{\nu} + m_{P}(3A^{2} + 3A + 1)\gamma^{\mu}\gamma^{\nu}.$$
(13)

The propagator is given by [10]

$$G^{\mu\nu} = \frac{1}{\gamma \cdot P_P - m_P} \left(-g^{\mu\nu} + \frac{1}{3} \gamma^{\mu} \gamma^{\nu} + \frac{1}{3m_P} (\gamma^{\mu} P_P^{\nu} - P_P^{\mu} \gamma^{\nu}) + \frac{2}{3m_P^2} P_P^{\mu} P_P^{\nu} \right) - \frac{1}{3m_P^2} \frac{A+1}{(2A+1)^2} \left((2A+1)(\gamma^{\mu} P_P^{\nu} + P_P^{\mu} \gamma^{\nu}) - \frac{A+1}{2} \gamma^{\mu} (\gamma \cdot P_P + 2m_P) \gamma^{\nu} + m \gamma^{\mu} \gamma^{\nu} \right).$$
(14)

To include interaction with photon, one uses the minimal substitution which guarantees gauge invariance to obtain the couplings. The lowest order interaction vertex $Q_P \bar{P}_{\alpha} \Gamma^{\alpha\beta}_{\mu} P_{\beta}$ which is different than spin-1/2 interaction vertex $Q_P \bar{P} \gamma_{\mu} P$. $\Gamma^{\alpha\beta}_{\mu}$ is given by

$$\gamma^{\mu}g_{\alpha\beta} + A(\gamma_{\alpha}g^{\mu}_{\beta} + g^{\mu}_{\alpha}\gamma_{\beta}) + \frac{1}{2}(3A^2 + 2A + 1)\gamma^{\alpha}\gamma_{\mu}\gamma^{\beta}.$$
(15)

The final result is A independent. In eq.12 we have chosen a particular case of A = 0 for simplicity. Therefore one should also use G_{α}^{ν} with A = 0 in eq.14.

III. NUMERICAL RESULTS

In our numerical studies, we will concentrate on processes involving pentaquarks with exotic quantum numbers, the Θ and $\Xi_{3/2}^{--}$. Processes involving other pentaquarks can be similarly carried out. We now display our numerical results for both spin-1/2 and spin-3/2, and different parities cases. For the pentaquark masses, we use $m_{\Theta} = 1542$ MeV and $m_{\Xi_{3/2}} = 1862$ MeV. We will treat the magnetic dipole moments as free parameters and let $\kappa_P = 2m_P\mu_P$ to vary between -1 to 1. The parameter $g_{PN\Pi}$ is determined by the decay width of the pentaquark. In our calculations we will express it as a function of Γ_{θ} .

A. Photoproduction

Photoproduction of pentaquark can provide useful information about the pentaquark properties[6]. An easy way of photoproduction of pentaqaurks is through a photon beam collides with a fixed target containing protons and neutrons. In this case, only production of Θ is possible via $\gamma n \to \Theta^+ K^-$, and $\gamma p \to \Theta^+ \bar{K}^0$. The results for the cross sections in the laboratory frame (fixed n and p) as functions of photon energies for both spin-1/2 and spin-3/2 are shown in Figs. 2 and 3.

FIG. 2: Cross sections for $\gamma n \to \Theta^+ K^-$ in the laboratory frame with spin 1/2 and 3/2. Figures a and b are for positive and negative parities, respectively



FIG. 3: Cross sections for $\gamma p \to \Theta^+ \bar{K}^0$ in the laboratory frame with spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



From Figs 2 and 3, it can be seen that for spin-1/2 case the cross section for $\gamma n \to \Theta^+ K^$ with positive parity has larger cross section than negative parity case. For example for $\kappa_{\Theta} = 0$ and $E_{\gamma} = 2.4$ GeV, the cross sections for these two cases are $155\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $17\Gamma(\Theta^+)nb \cdot MeV^{-1}$, respectively. The cross section for $\gamma p \to \Theta^+ K^0$ with positive parity has larger cross section than negative parity case, the cross sections for these two cases are $47\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $18\Gamma(\Theta^+)nb \cdot MeV^{-1}$, respectively.

For spin-3/2, the negative parity case has larger cross section compared with positive parity case. For example with $\kappa_{\Theta} = 0$ and $E_{\gamma} = 2.4$ GeV, the cross sections for $\gamma n \to \Theta^+ K^-$ are $2350\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $691\Gamma(\Theta^+)nb \cdot MeV^{-1}$ for negative parity and positive parity. The cross sections for $\gamma p \to \Theta^+ K^0$ are $1953\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $184\Gamma(\Theta^+)nb \cdot MeV^{-1}$ respectively.

One can clearly see from Figures 2 and 3 that regardless the parity, spin-3/2 pentaquark has cross section larger than spin-1/2. This can provide important information about the spin. The separation between the cross sections with positive and negative parities is large which can be used to obtain information about the parity of the pentaqaurk too.

The cross sections also depend on magnetic dipole moment of pentaquarks. From the figures we see that the changes in the cross section can vary several times when κ changes from -1 to 1.

The case for Θ with spin-1/2 has been discussed in Ref.[5, 6]. Our approach is the same as that used in Ref.[5] and we agree with their results which are shown in Fig. 2. Our approach is different than that used in Ref.[6]. This leads to the different behavior of photon energy E_{γ} dependence. Detailed experimental data will provide more information about the underlying theory for photoproduction. In our estimate we have neglected other possible intermediate states, such as K^* which can change the cross section. But model calculations show that K^* contribution does not change the general features[6]. We expect that the results obtained here provide a reasonable estimate.

B. Radiative Decays

Once pentaquarks are produced they can decay radiatively through $\Theta^+ \to \gamma K^+ n$, $\Theta^+ \to \gamma K^0 p$, and $\Xi_{3/2}^{--} \to \gamma K^- \Sigma^-$, $\Xi_{3/2}^{--} \to \gamma \pi^- \Xi^-$, respectively.

It is well known that there are divergencies when photon energies approach zero in radiative decays of the types discussed here. To remedy these divergencies, we require that the photon energies to be larger than 0.05 MeV. The results for radiative Θ decays are shown in Figs. 4 and 5. The results for radiative $\Xi_{3/2}^{--}$ decays are shown in Figs. 6 and 7.

FIG. 4: Radiative $\Theta^+ \to \gamma n K^+$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



FIG. 5: Radiative $\Theta^+ \to \gamma p \bar{K}^0$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



For Θ^+ radiative decays, the branching ratios for spin-1/2 and spin-3/2 cases are approximately 1.3×10^{-3} and 4×10^{-4} for $\Theta^+ \to \gamma n K^+$ and $\Theta^+ \to \gamma p K^0$, respectively. These can be used to check the consistence of the model. However, the branching ratios for these decays are not sensitive to the spin, parity and anomalous magnetic dipole moment of the pentaqaurks.

The situation changes when consider radiative decays of Ξ^{--} . From Figs. 6 and 7, one can see that the branching ratios for spin-1/2 cases are about two times larger than the

branching ratios for spin-3/2 cases. It is also interesting to note that the branching ratio for $\Xi^{--} \rightarrow \gamma \Xi^{-} \pi^{-}$ is at the level of a few percent which may be easily studied experimentally.

FIG. 6: Radiative $\Xi_{3/2}^{--} \to \gamma \Sigma^- K^-$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



FIG. 7: Radiative $\Xi_{3/2}^{--} \rightarrow \gamma \Xi^- \pi^-$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



In conclusion we have studied several radiative processes of pentaquarks using chiral perturbatin theory. We find that the photoproduction cross sections of Θ^+ are sensitive to the spin, parity and anomalous magnetic dipole moment of the pentaquark. Radiative decays of Θ^+ can also provide consistent check of the theory although these decays are not very sensitive to the spin, parity and anomalous magnetic dipole moment. Radiative decays

of Ξ^{--} are sensitive to the spin of the pentaquark. Near future experiments on pentaquark radiative processes can provide important information about pentaquark properties.

- [1] T. Nakano et al, [LEPS Collaboration], Phys. Rev. Lett. 91, 012002(2003)[arXiv:hep-ex/0301020].
- [2] C. Alt et al. [NA49 Collaboration], arXiv:hep-ex/0310014.
- [3] For a recent review on experimental data see for example, Ken Hicks, arXiv: hep-ph/0408001;
 S. Jin, Plenary talk presented at ICHEP'04, Aug. 16 22, 2004, Beijing, China.
- [4] A.V. Manohar, Nucl. Phys. B248, 19(1984); M. Chemtob, Nucl. Phys. B256, 600(1985);
 M. Praszalowicz, in Skrmions and Anomalies, M. Jezabeck and M. Praszolowicz, eds. World Scientific, 112(1987); Phys. Lett. B575, 234(2003)[arXiv:hep-ph/0308114]; Y. Oh, Phys. Lett. B331, 362(1994); M.-L. Yan and X.-H. Meng, Commun. Theor. Phys. 24, 435(1995); D. Diakonov. V. Petrov and M. V. Polyakov, Z. Phys. A359, 305(1997).
- [5] P. Ko, Jungil Lee, Taekoon Lee, Jae-hyeon Park, arXiv: hep-ph/0312147.
- [6] Byung Geel Yu, Tae Keun Choi, Chueng-Ryong Ji, arXiv: nucl-th/0408006; Frank E. Close, Qiang Zhao, Phys. Lett. B590:176-184,2004, arXiv: hep-ph/0403159; Seung-Il Nam, Atsushi Hosaka, Hyun-Chul Kim, arXiv: hep-ph/0403009; Michail P. Rekalo, Egle Tomasi-Gustafsson, arXiv: hep-ph/0401050; M.V. Polyakov and Rahtke, Eur. J. A18, 691(2003); Y. Oh, H. Kim and S.H. Lee, arXiv: hep-ph/031229; S.H. Lee, H. Kim and Y. Oh, arXiv: hep-ph/0402135; W. Liu, C.M. Ko, Nucl. Phys. A741:215-233,2004, arXiv: nucl-th/0309023; W. Liu, C.M. Ko, Phys.Rev.C68:045203,2003, arXiv: nucl-th/0308034; Hai-Yan Gao, Bo-Qiang Ma, Mod. Phys. Lett.A14:2313-2319,1999, arXiv: hep-ph/0305294.
- Thomas Mehen, Carlos Schat, Phys.Lett.B588:67-73,2004, arXiv: hep-ph/0401107; T. Inoue,
 V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, arXiv: hep-ph/0407305; W.W. Li, et al.,
 arXiv: hep-ph/0312362.
- [8] Hyun-Chul Kim, Michal Praszalowicz, Phys.Lett.B585:99-105,2004, arXiv: hep-ph/0405171;
 Y.R. Liu et al., Phys.Rev.C69:035205,2004, arXiv: hep-ph/0312074; Peng-Zhi Huang, Wei-Zhen Deng, Xiao-Lin Chen, Shi-Lin Zhu, Phys.Rev.D69:074004,2004, arXiv: hep-ph/0311108;
 T. Inoue, V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, arXiv: hep-ph/0408057;
 R. Bijker, M.M. Giannini, E. Santopinto, Phys.Lett.B595:260-268,2004, [HEP-PHarXiv:

hep-ph/0403029.

- [9] For recent reviews see for exaple, Shi-Lin Zhu, arXiv: hep-ph/0406204; Makoto Oka, arXiv: hep-ph/0406211; F. Close Plenary talk presented at ICHEP'04, Aug. 16 22, 2004, Beijing, China.
- [10] H. Haberzettl, arXiv: nucl-th/9812043; M. Benmerrouche, R.M. Davidson and M. C. Mukhopadhyay, Phys. Rev. C39, 2339(1989); L.M. Nath, B. Etemadi and J. Kimel, Phys. Rev. D3, 2153(1971); A. Aurilia and H. Umezawa, Phys. Rev. 128, 1682(1969); C. Fronsdal, Nuov. Cimento Suppl. 9, 416(1958); P.A. Moldauer and K. Case, Phys. Rev. 102, 279(1956).

Photoproduction and Radiative Decay of Spin 1/2 and 3/2 Pentaquarks

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Abstract

We study photoproduction and radiative decays of pentauqarks paying particular attention to the differences between spin-1/2 and spin-3/2, positive and negative parities of pentaquarks. Detailed study of these processes can not only give crucial information about the spin, but also the parity of pentaquarks.

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I. INTRODUCTION

Recently several experiments have reported evidences for pentaquarks Θ and other states[1–3]. The first observed pentaquark state was the $\Theta(1540)$ with strangeness S = +1and was identified as a state with quark content $udud\bar{s}$. This particle is an isosinglet and belongs to the anti-decuplet multiplet in flavor $SU(3)_f$ symmetry[4]. Consequently NA49 has reported evidences for isoquated $\Xi_{3/2}$ in the anit-decuplet[2]. At present there are very limited information on the detailed properties such as the spin, the parity and the magnetic dipole moment. Several other experiments have also carried out searches for these particles. Some of them reported positive and while others reported negative results[3]. One has to wait future experiments to decide whether these pentaquark state are real. On the theoretical front, there are also many studies trying to understand the propertices of these possible pentaquark states[5–9]

In this paper we explore possibilities of studying the properties of pentaquark Θ and its partners in the SU(3) anti-decuplet multiplet, using radiative processes involving a pentaquark P, an ordinary baryon N and a pseudoscalar Π . We consider two classes of processes, the photoproduction $\gamma + N \to \Pi P$ and radiative decay $P \to N \Pi \gamma$.

In the above N and Π indicate a member in the ordinary baryon octet and pseudoscalar octet of $SU(3)_f$, respectively. They are given by

$$N = (N_i^j) = \begin{pmatrix} \frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \ \Sigma^+ & p \\ \Sigma^- & -\frac{\Sigma^0}{\sqrt{2}} + \frac{\Lambda}{\sqrt{6}} \ n \\ \Xi^- & \Xi^0 & -\frac{2\Lambda}{\sqrt{6}} \end{pmatrix}, \quad \Pi = (\Pi_i^j) = \begin{pmatrix} \frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \ \pi^+ & K^+ \\ \pi^- & -\frac{\pi^0}{\sqrt{2}} + \frac{\eta}{\sqrt{6}} \ K^0 \\ K^- & \bar{K}^0 & -\frac{2\eta}{\sqrt{6}} \end{pmatrix} (1)$$

P is a member of the anti-decuplet ($\overline{10}$) pentaquark multiplet. This multiplet has 10 members which can be described by a totally symmetric tensor P^{ijk} in SU(3). The 10 members are

$$P^{111} = \Xi_{3/2}^{--}, \quad P^{112} = \Xi_{3/2}^{-}/\sqrt{3}, \quad P^{122} = \Xi_{3/2}^{0}/\sqrt{3}, \quad P^{222} = \Xi_{3/2}^{+},$$

$$P^{113} = \Sigma_{a}^{-}/\sqrt{3}, \quad P^{123} = \Sigma_{a}^{0}/\sqrt{6}, \quad P^{223} = \Sigma_{a}^{+}/\sqrt{3},$$

$$P^{133} = N_{a}^{0}/\sqrt{3}, \quad P^{233} = N_{a}^{+}/\sqrt{3}, \quad P^{333} = \Theta^{+}.$$
(2)

Without $SU(3)_f$ symmetry breaking members in a $SU(3)_f$ multiplet all have the same

mass. The degeneracy of mass is lifted by the light quark mass differences, m_u , m_d and m_s . Using information on the masses of Θ and $\Xi_{3/2}$ including the leading $SU(3)_f$ breaking effects, the masses of the anti-decuplet members are given by[5] $m_{\Theta} = 1542$ MeV, $m_{\Xi_{3/2}} = 1862$ MeV, $m_{\Sigma_a} = 1755$ MeV, and $m_{N_a} = 1648$ MeV.

Discussions for radiative processes involving a P, a N, a Π and a γ with spin-1/2 pentaquarks have been carried out in several papers[5, 6]. There are also some studies for spin-3/2 pentaquarks[7], but no detailed studies of radiative processes. In this work we will consider both spin-1/2 and spin-3/2 cases and paying particular attention for the differences. Since in the processes considered involve pseudoscalar goldstone bosons π and K, we will use chiral perturbation theory to carry out the analysis.

II. THE MATRIX ELEMENTS FOR RADIATIVE PROCESSES

The leading order diagrams for the radiative processes involving a P, a N, a Π and a γ are shown in Figure 1. The electromagnetic coupling of photon with Π and N are known. To evaluate these diagrams, we need to know the various couplings involving pentaquarks.

A. The spin-1/2 case

There are two types of electromagnetic couplings, the electric charge and magnetic dipole interactions. The leading chiral electric charge and magnetic dipole couplings are given by

$$L_{e} = \bar{P}i\gamma^{\mu}D_{\mu}P = \bar{P}_{ijk}i\gamma^{\mu}(\partial_{\mu}P^{ijk} - V^{i}_{\mu,l}P^{ljk} - V^{j}_{\mu,l}P^{ilk} - V^{k}_{\mu,l}P^{ijl}),$$

$$L_{m} = \frac{\mu_{P}}{4}\bar{P}_{ijk}\sigma^{\mu\nu}(f^{i}_{\mu\nu,l}P^{ljk} + f^{j}_{\mu\nu,l}P^{ilk} + f^{k}_{\mu\nu,l}P^{ijl}),$$
(3)

where $V_{\mu} = (1/2)(\xi^{\dagger}\partial_{\mu}\xi + \xi\partial_{\mu}\xi^{\dagger}) + i(e/2)A_{\mu}(\xi^{\dagger}Q\xi + \xi Q\xi^{\dagger})$. Here $\xi = exp[i\Pi/\sqrt{2}f_{\pi}]$ and Q = Diag(2/3, -1/3, -1/3) is the quark charge matrix and A_{μ} is the photon field. $f_{\mu\nu,i}^{j} = F_{\mu\nu}(\xi^{\dagger}Q\xi + \xi Q\xi^{\dagger})_{i}^{j}$ with $F_{\mu\nu}$ being the photon field strength. Expanding to the leading order, we have for each individual pentaquark

$$L_e = -eQ_i \bar{P}_i \gamma^{\mu} A_{\mu} P_i,$$

$$L_m = -\frac{e\mu_P Q_i}{2} \bar{P}_i \sigma^{\mu\nu} F_{\mu\nu} P_i.$$
(4)

We note that for neutral pentuaquarks, to the leading order the anomalous dipole moments are zero. The kappa parameter $\kappa_P = 2m_P\mu_P$ have been estimated to be of order one[8]. In our analysis we will treat it as a free parameter to see if experimental data can provide some information.

We also need to know the strong interaction coupling of a pentaquark with an ordinary baryon and a pseudoscalar. It can be parameterized as

$$L_{PN\Pi} = g_{PN\Pi} \bar{P}_{ilm} \Gamma_P \gamma^{\mu} (\tilde{A}_{\mu})^l_j N^m_k \epsilon^{ijk} + H.C.$$
(5)

In the above Γ_p takes "+1" and " γ_5 " if P has negative and positive parities, respectively. $\tilde{A}_{\mu} = (i/2)(\xi^{\dagger}\partial_{\mu}\xi - \xi\partial_{\mu}\xi^{\dagger}) - (e/2)A_{\mu}(\xi^{\dagger}Q\xi - \xi Q\xi^{\dagger}).$

Expanding the above effective Lagrangian to the leading order we obtain $P - N - \Pi$ type of couplings. The results are given in Table 1.





Θ^+	$-nK^+ + pK^0$
N_a^0	$\frac{1}{6}(-3\sqrt{2}n\eta + 3\sqrt{2}\Lambda K^0 + \sqrt{6}\Sigma^0 K^0 - \sqrt{6}n\pi^0 + 2\sqrt{3}p\pi^ 2\sqrt{3}\Sigma_a^- K^+)$
N_a^+	$\frac{1}{6}(3\sqrt{2}p\eta - 3\sqrt{2}\Lambda K^{+} + \sqrt{6}\Sigma^{0}K^{+} - \sqrt{6}p\pi^{0} - 2\sqrt{3}n\pi^{+} + 2\sqrt{3}\Sigma^{+}K^{0})$
Σ_a^-	$\frac{1}{6}(2\sqrt{3}nK^{-} + 3\sqrt{2}\Lambda\pi^{-} + \sqrt{6}\Sigma^{0}\pi^{-} - 3\sqrt{2}\Sigma^{-}\eta - \sqrt{6}\Sigma^{-}\pi^{0} - 2\sqrt{3}\Xi^{-}K^{0})$
Σ_a^0	$\frac{1}{6}(\sqrt{6}n\bar{K}^0 - \sqrt{6}pK^ 3\sqrt{2}\Lambda\pi^0 + 3\sqrt{2}\Sigma^0\eta - \sqrt{6}\Sigma^-\pi^+ + \sqrt{6}\Sigma^+\pi^ \sqrt{6}\Xi^0K^0 + \sqrt{6}\Xi^-K^+)$
Σ_a^+	$\frac{1}{6}(-2\sqrt{3}p\bar{K}^0 - 3\sqrt{2}\Lambda\pi^+ + \sqrt{6}\Sigma^0\pi^+ + 3\sqrt{2}\Sigma^+\eta - \sqrt{6}\Sigma^+\pi^0 + 2\sqrt{3}\Xi^0K^+)$
$\Xi_{3/2}^{}$	$\Sigma^- K^ \Xi^- \pi^-$
$\Xi^{3/2}$	$\frac{1}{6}(-2\sqrt{6}\Sigma^0 K^- + 2\sqrt{3}\Sigma^- \bar{K}^0 - 2\sqrt{3}\Xi^0 \pi^- + 2\sqrt{6}\Xi^- \pi^0)$
$\Xi_{3/2}^{0}$	$\frac{1}{6}(-2\sqrt{6}\Sigma^0\bar{K}^0 - 2\sqrt{3}\Sigma^+K^- + 2\sqrt{3}\Xi^0\pi^0 + 2\sqrt{6}\Xi^-\pi^+)$
$\Xi^{+}_{3/2}$	$-\Sigma^+ \bar{K}^0 + \Xi^0 \pi^+$

TABLE I: P-N- Π couplings in unit $g_{PN\Pi}/\sqrt{2}f_{\pi}$. The couplings in the tables are understood to be in the form $-a_{PN\Pi}\bar{P}\Gamma_P\gamma^{\mu}N\partial_{\mu}\Pi$. The coefficient in front of $N\Pi$ in the second column is $-a_{PN\Pi}$.

The contact γ -P-N- Π coupling in Figure 1.d is obtained from a term $ieg_{PN\Pi}A_{\mu}\bar{P}_{ilm}\Gamma_{P}\gamma^{\mu}[\Pi,Q]_{j}^{l}N_{k}^{m}\epsilon^{ijk}$ obtained by expanding $L_{PN\Pi}$.

In the following we display the matrix element for $P \to N\Pi\gamma$. The matrix element for $\gamma N \to P\Pi$ can be obtained by making appropriate changes of signs for the relevant particle momenta. We have

$$M(P \to N\Pi\gamma) = \frac{eg_{PN\Pi}}{\sqrt{2}f} a_{PN\Pi} \epsilon_{\mu}^* \bar{N} [Q_{\Pi} \Gamma_P \gamma^{\mu} - (Q_N \gamma^{\mu} + \frac{\mu_N}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) \frac{1}{\gamma \cdot P_{\gamma} + \gamma \cdot P_N - m_N} \Gamma_P \gamma \cdot P_{\pi} - \Gamma_P \gamma \cdot P_{\Pi} \frac{1}{\gamma \cdot P_N + \gamma \cdot P_{\Pi} - m_P} (Q_P \gamma^{\mu} + \frac{\mu_P}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) - Q_{\Pi} \frac{(2P_{\Pi} + P_{\gamma})^{\mu}}{(P_{\Pi} + P_{\gamma})^2 - m_{\Pi}^2} \Gamma_P (\gamma \cdot P_{\Pi} + \gamma \cdot P_{\gamma})] P.$$
(6)

For $\Theta^+ \to nK^+\gamma$, $a_{PN\Pi} = a_{\Theta nK} = 1$, $Q_P = Q_\Theta = 1$, $Q_N = Q_n = 0$, $Q_{\Pi} = Q_{K^+} = 1$. For $\Theta^+ \to pK^0\gamma$, $a_{PN\Pi} = a_{\Theta pK} = -1$, $Q_N = Q_p = 1$ and $Q_{K^0} = 0$. For $\Xi_{3/2}^{--} \to \Sigma^- K^-\gamma$, $a_{PN\Pi} = a_{\Xi_{3/2}^{--}\Sigma^-K^-} = -1$, $Q_P = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Sigma^-} = -1$, $Q_{\Pi} = Q_{K^-} = -1$. And for $\Xi_{3/2}^{--} \to \Xi^-\pi^-\gamma$, $a_{PN\Pi} = a_{\Xi_{3/2}^{--}\Xi^-\pi^-} = 1$, $Q_P = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -2$, $Q_N = Q_{\Xi_{3/2}^{--}} = -1$, $Q_{\Pi} = Q_{\pi^-} = -1$.

The parameter $g_{PN\Pi}$ can be determined from a pentaquark P decays into a baryon and

a meson. For example

$$\frac{g_{PN\Pi}^2}{2f_{\pi}^2} = \frac{\Gamma(\Theta^+ \to nK^+) 16\pi m_{\Theta}}{(m_n + \hat{P}m_{\Theta})^2 ((m_n - \hat{P}m_{\Theta})^2 - m_K^2) Phase},$$

$$Phase = \sqrt{(1 - (m_K + m_n)^2/m_{\Theta}^2)((1 - (m_K - m_n)^2/m_{\Theta}^2))}.$$
(7)

In the above " \hat{P} " is the eigenvalue of the parity, it takes "+" for positive parity and "-" for negative parity pentaquark, respectively.

From Table 1 we see that Θ^+ only has two strong decay channels, pK^0 and nK^+ . The total width of Θ^+ is therefore $\Gamma_{\Theta} = \Gamma(\Theta^+ \to pK^0) + \Gamma(\Theta^+ \to nK^+)$. If the Γ_{Θ} is determined, one can determine $g_{PN\pi}^2$ from eq.7

B. The spin-3/2 case

In this case one needs to use the Rarita-Schwinger field for pentaquarks P_{ilm}^{μ} . The electromagnetic couplings needed are modified compared with spin-1/2 particles, and they are given by

$$L_{e} = \bar{P}^{\alpha} i \gamma^{\mu} D_{\mu} P_{\alpha} = \bar{P}^{\alpha}_{ijk} i \gamma^{\mu} (\partial_{\mu} P^{ijk}_{\alpha} - V^{i}_{\mu,l} P^{ljk}_{\alpha} - V^{j}_{\mu,l} P^{ilk}_{\alpha} - V^{k}_{\mu,l} P^{ijl}_{\alpha}),$$

$$L_{m} = \frac{\mu_{P}}{4} \bar{P}^{\alpha}_{ijk} \sigma^{\mu\nu} (f^{i}_{\mu\nu,l} P^{ljk}_{\alpha} + f^{j}_{\mu\nu,l} P^{ilk}_{\alpha} + f^{k}_{\mu\nu,l} P^{ijl}_{\alpha}).$$
(8)

Since a spin-3/2 particle can have dipole and quadrupole moments, if both are not zero, one should add another term to the electromagnetic couplings,

$$L_q = \tau_P \bar{P}_\nu F^{\mu\nu} P_\mu, \tag{9}$$

We will take it to be zero in our later discussions.

The chiral Lagrangian for strong coupling involving a pentaquark, a baryon and a pseudoscalar is given by

$$L_{PN\Pi} = g_{PN\Pi} \bar{P}^{\mu}_{ilm} \gamma_5 \Gamma_P (A_{\mu})^l_j N^m_k \epsilon^{ijk} + H.C.$$
(10)

From the above we have

$$\Gamma(P \to N\Pi) = \frac{g_{PN\Pi}^2}{2f^2} \frac{Phase}{16\pi m_P} \frac{1}{3} ((\hat{P}m_P + m_N)^2 - m_{\Pi}^2) \\ \times (\frac{1}{4m_P^2} (m_P^2 + m_{\Pi}^2 - m_N^2)^2 - m_{\Pi}^2).$$
(11)

Combining the above information we obtain the matrix element for $P \to N \Pi \gamma$

$$M(P \to N\Pi\gamma) = \frac{eg_{PN\Pi}}{\sqrt{2}f} a_{PN\Pi} \epsilon_{\mu}^{*} \bar{N} [Q_{\Pi}\gamma_{5}\Gamma_{P}g^{\mu\nu} - (Q_{N}\gamma^{\mu} + \frac{\mu_{N}}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) \frac{1}{\gamma \cdot P_{\gamma} + \gamma \cdot P_{N} - m_{N}} \gamma_{5}\Gamma_{P}P_{\pi}^{\nu} + \gamma_{5}\Gamma_{P}P_{\Pi}^{\alpha}G_{\alpha}^{\ \nu}(Q_{P}\gamma^{\mu} + \frac{\mu_{P}}{2} [\gamma^{\mu}, \gamma \cdot P_{\gamma}]) - Q_{\Pi} \frac{(2P_{\Pi} + P_{\gamma})^{\mu}}{(P_{\Pi} + P_{\gamma})^{2} - m_{\Pi}^{2}} \gamma_{5}\Gamma_{P}(P_{\Pi} + P_{\gamma})^{\nu}]P_{\nu}.$$
(12)

In the above $G^{\mu\nu}$ is the spin-3/2 propagator resulting from the following most general Lagrangian[10]

$$L = \bar{P}_{\mu}\Lambda^{\mu\nu}P_{\nu},$$

$$\Lambda^{\mu\nu} = (\gamma \cdot P_{P} - m_{P})g^{\mu\nu} + A(\gamma^{\mu}P_{P}^{\nu} + P_{P}^{\mu}\gamma^{\nu})$$

$$+ \frac{1}{2}(3A^{2} + 2A + 1)\gamma^{\mu}\gamma \cdot P_{P}\gamma^{\nu} + m_{P}(3A^{2} + 3A + 1)\gamma^{\mu}\gamma^{\nu}.$$
(13)

The propagator is given by[10]

$$G^{\mu\nu} = \frac{1}{\gamma \cdot P_P - m_P} \left(-g^{\mu\nu} + \frac{1}{3} \gamma^{\mu} \gamma^{\nu} + \frac{1}{3m_P} (\gamma^{\mu} P_P^{\nu} - P_P^{\mu} \gamma^{\nu}) + \frac{2}{3m_P^2} P_P^{\mu} P_P^{\nu} \right) - \frac{1}{3m_P^2} \frac{A+1}{(2A+1)^2} \left((2A+1)(\gamma^{\mu} P_P^{\nu} + P_P^{\mu} \gamma^{\nu}) - \frac{A+1}{2} \gamma^{\mu} (\gamma \cdot P_P + 2m_P) \gamma^{\nu} + m \gamma^{\mu} \gamma^{\nu} \right).$$
(14)

To include interaction with photon, one uses the minimal substitution which guarantees gauge invariance to obtain the couplings. The lowest order interaction vertex $Q_P \bar{P}_{\alpha} \Gamma^{\alpha\beta}_{\mu} P_{\beta}$ which is different than spin-1/2 interaction vertex $Q_P \bar{P} \gamma_{\mu} P$. $\Gamma^{\alpha\beta}_{\mu}$ is given by

$$\gamma^{\mu}g_{\alpha\beta} + A(\gamma_{\alpha}g^{\mu}_{\beta} + g^{\mu}_{\alpha}\gamma_{\beta}) + \frac{1}{2}(3A^2 + 2A + 1)\gamma^{\alpha}\gamma_{\mu}\gamma^{\beta}.$$
 (15)

The final result is A independent. In eq.12 we have chosen a particular case of A = 0 for simplicity. Therefore one should also use G_{α}^{ν} with A = 0 in eq.14.

III. NUMERICAL RESULTS

In our numerical studies, we will concentrate on processes involving pentaquarks with exotic quantum numbers, the Θ and $\Xi_{3/2}^{--}$. Processes involving other pentaquarks can be similarly carried out. We now display our numerical results for both spin-1/2 and spin-3/2, and different parities cases. For the pentaquark masses, we use $m_{\Theta} = 1542$ MeV and $m_{\Xi_{3/2}} = 1862$ MeV. We will treat the magnetic dipole moments as free parameters and let $\kappa_P = 2m_P\mu_P$ to vary between -1 to 1. The parameter $g_{PN\Pi}$ is determined by the decay width of the pentaquark. In our calculations we will express it as a function of Γ_{θ} .

A. Photoproduction

Photoproduction of pentaquark can provide useful information about the pentaquark properties[6]. An easy way of photoproduction of pentaqaurks is through a photon beam collides with a fixed target containing protons and neutrons. In this case, only production of Θ is possible via $\gamma n \to \Theta^+ K^-$, and $\gamma p \to \Theta^+ \bar{K}^0$. The results for the cross sections in the laboratory frame (fixed n and p) as functions of photon energies for both spin-1/2 and spin-3/2 are shown in Figs. 2 and 3.

FIG. 2: Cross sections for $\gamma n \to \Theta^+ K^-$ in the laboratory frame with spin 1/2 and 3/2. Figures a and b are for positive and negative parities, respectively



FIG. 3: Cross sections for $\gamma p \to \Theta^+ \bar{K}^0$ in the laboratory frame with spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



From Figs 2 and 3, it can be seen that for spin-1/2 case the cross section for $\gamma n \to \Theta^+ K^$ with positive parity has larger cross section than negative parity case. For example for $\kappa_{\Theta} = 0$ and $E_{\gamma} = 2.4$ GeV, the cross sections for these two cases are $155\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $17\Gamma(\Theta^+)nb \cdot MeV^{-1}$, respectively. The cross section for $\gamma p \to \Theta^+ K^0$ with positive parity has larger cross section than negative parity case, the cross sections for these two cases are $47\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $18\Gamma(\Theta^+)nb \cdot MeV^{-1}$, respectively.

For spin-3/2, the negative parity case has larger cross section compared with positive parity case. For example with $\kappa_{\Theta} = 0$ and $E_{\gamma} = 2.4$ GeV, the cross sections for $\gamma n \to \Theta^+ K^-$ are $2350\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $691\Gamma(\Theta^+)nb \cdot MeV^{-1}$ for negative parity and positive parity. The cross sections for $\gamma p \to \Theta^+ K^0$ are $1953\Gamma(\Theta^+)nb \cdot MeV^{-1}$ and $184\Gamma(\Theta^+)nb \cdot MeV^{-1}$ respectively.

One can clearly see from Figures 2 and 3 that regardless the parity, spin-3/2 pentaquark has cross section larger than spin-1/2. This can provide important information about the spin. The separation between the cross sections with positive and negative parities is large which can be used to obtain information about the parity of the pentaquark too.

The cross sections also depend on magnetic dipole moment of pentaquarks. From the figures we see that the changes in the cross section can vary several times when κ changes from -1 to 1.

The case for Θ with spin-1/2 has been discussed in Ref.[5, 6]. Our approach is the same as that used in Ref.[5] and we agree with their results which are shown in Fig. 2. Our approach is different than that used in Ref.[6]. This leads to the different behavior of photon energy E_{γ} dependence. Detailed experimental data will provide more information about the underlying theory for photoproduction. In our estimate we have neglected other possible intermediate states, such as K^* which can change the cross section. But model calculations show that K^* contribution does not change the general features[6]. We expect that the results obtained here provide a reasonable estimate.

B. Radiative Decays

Once pentaquarks are produced they can decay radiatively through $\Theta^+ \to \gamma K^+ n$, $\Theta^+ \to \gamma K^0 p$, and $\Xi_{3/2}^{--} \to \gamma K^- \Sigma^-$, $\Xi_{3/2}^{--} \to \gamma \pi^- \Xi^-$, respectively.

It is well known that there are divergencies when photon energies approach zero in radiative decays of the types discussed here. To remedy these divergencies, we require that the photon energies to be larger than 0.05 MeV. The results for radiative Θ decays are shown in Figs. 4 and 5. The results for radiative $\Xi_{3/2}^{--}$ decays are shown in Figs. 6 and 7.

1.360 1.3690 (b) 1.358 (a) 1.356 1.3685 1.354 3 $P(\theta)=$ 1.352 1.3680 10^{3} BR(θ^{+} ->nK⁺ γ) ->nK S(*θ*)=1/2 . 1.350 -S(θ)=3/2 1.348 1.3675 1.346 ${}^{3}\mathsf{BR}(\theta^{+})$ 1.344 1.3670 1.342 1.340 1.3665 1.338 <u>10</u> 1.336 S(0)=1/2 1.3660 1.334 $S(\theta)=3/2$ 1.3655 1.332 1.330 1.328 -1.3650 -0.5 0.0 0.5 -0.5 0.0 0.5 1.0 -1.0 1.0 -1.0 K_{θ} K_{θ}

FIG. 4: Radiative $\Theta^+ \to \gamma n K^+$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively

FIG. 5: Radiative $\Theta^+ \to \gamma p \bar{K}^0$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



For Θ^+ radiative decays, the branching ratios for spin-1/2 and spin-3/2 cases are approximately 1.3×10^{-3} and 4×10^{-4} for $\Theta^+ \to \gamma n K^+$ and $\Theta^+ \to \gamma p K^0$, respectively. These can be used to check the consistence of the model. However, the branching ratios for these decays are not sensitive to the spin, parity and anomalous magnetic dipole moment of the pentaqaurks.

The situation changes when consider radiative decays of Ξ^{--} . From Figs. 6 and 7, one can see that the branching ratios for spin-1/2 cases are about two times larger than the

branching ratios for spin-3/2 cases. It is also interesting to note that the branching ratio for $\Xi^{--} \rightarrow \gamma \Xi^{-} \pi^{-}$ is at the level of a few percent which may be easily studied experimentally.

FIG. 6: Radiative $\Xi_{3/2}^{--} \rightarrow \gamma \Sigma^- K^-$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



FIG. 7: Radiative $\Xi_{3/2}^{--} \rightarrow \gamma \Xi^- \pi^-$ decay for spin-1/2 and spin-3/2. Figures a and b are for positive and negative parities, respectively



In conclusion we have studied several radiative processes of pentaquarks using chiral perturbatin theory. We find that the photoproduction cross sections of Θ^+ are sensitive to the spin, parity and anomalous magnetic dipole moment of the pentaquark. Radiative decays of Θ^+ can also provide consistent check of the theory although these decays are not very sensitive to the spin, parity and anomalous magnetic dipole moment. Radiative decays

of Ξ^{--} are sensitive to the spin of the pentaquark. Near future experiments on pentaquark radiative processes can provide important information about pentaquark properties.

- T. Nakano et al, [LEPS Collaboration], Phys. Rev. Lett. 91, 012002(2003)[arXiv:hepex/0301020].
- [2] C. Alt et al. [NA49 Collaboration], arXiv:hep-ex/0310014.
- [3] For a recent review on experimental data see for example, Ken Hicks, arXiv: hep-ph/0408001;
 S. Jin, Plenary talk presented at ICHEP'04, Aug. 16 22, 2004, Beijing, China.
- [4] A.V. Manohar, Nucl. Phys. B248, 19(1984); M. Chemtob, Nucl. Phys. B256, 600(1985);
 M. Praszalowicz, in Skrmions and Anomalies, M. Jezabeck and M. Praszolowicz, eds. World Scientific, 112(1987); Phys. Lett. B575, 234(2003)[arXiv:hep-ph/0308114]; Y. Oh, Phys. Lett. B331, 362(1994); M.-L. Yan and X.-H. Meng, Commun. Theor. Phys. 24, 435(1995); D. Diakonov. V. Petrov and M. V. Polyakov, Z. Phys. A359, 305(1997).
- [5] P. Ko, Jungil Lee, Taekoon Lee, Jae-hyeon Park, arXiv: hep-ph/0312147.
- [6] Byung Geel Yu, Tae Keun Choi, Chueng-Ryong Ji, arXiv: nucl-th/0408006; Frank E. Close, Qiang Zhao, Phys. Lett. B590:176-184,2004, arXiv: hep-ph/0403159; Seung-Il Nam, Atsushi Hosaka, Hyun-Chul Kim, arXiv: hep-ph/0403009; Michail P. Rekalo, Egle Tomasi-Gustafsson, arXiv: hep-ph/0401050; M.V. Polyakov and Rahtke, Eur. J. A18, 691(2003); Y. Oh, H. Kim and S.H. Lee, arXiv: hep-ph/031229; S.H. Lee, H. Kim and Y. Oh, arXiv: hep-ph/0402135; W. Liu, C.M. Ko, Nucl. Phys. A741:215-233,2004, arXiv: nucl-th/0309023; W. Liu, C.M. Ko, Phys.Rev.C68:045203,2003, arXiv: nucl-th/0308034; Hai-Yan Gao, Bo-Qiang Ma, Mod. Phys. Lett.A14:2313-2319,1999, arXiv: hep-ph/0305294.
- Thomas Mehen, Carlos Schat, Phys.Lett.B588:67-73,2004, arXiv: hep-ph/0401107; T. Inoue,
 V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, arXiv: hep-ph/0407305; W.W. Li, et al.,
 arXiv: hep-ph/0312362.
- [8] Hyun-Chul Kim, Michal Praszalowicz, Phys.Lett.B585:99-105,2004, arXiv: hep-ph/0405171;
 Y.R. Liu et al., Phys.Rev.C69:035205,2004, arXiv: hep-ph/0312074; Peng-Zhi Huang, Wei-Zhen Deng, Xiao-Lin Chen, Shi-Lin Zhu, Phys.Rev.D69:074004,2004, arXiv: hep-ph/0311108;
 T. Inoue, V.E. Lyubovitskij, Th. Gutsche, Amand Faessler, arXiv: hep-ph/0408057; R. Bijker, M.M. Giannini, E. Santopinto, Phys.Lett.B595:260-268,2004, [HEP-PHarXiv: hep-Ph/0408057]

ph/0403029.

- [9] For recent reviews see for exaple, Shi-Lin Zhu, arXiv: hep-ph/0406204; Makoto Oka, arXiv: hep-ph/0406211; F. Close Plenary talk presented at ICHEP'04, Aug. 16 22, 2004, Beijing, China.
- [10] H. Haberzettl, arXiv: nucl-th/9812043; M. Benmerrouche, R.M. Davidson and M. C. Mukhopadhyay, Phys. Rev. C39, 2339(1989); L.M. Nath, B. Etemadi and J. Kimel, Phys. Rev. D3, 2153(1971); A. Aurilia and H. Umezawa, Phys. Rev. 128, 1682(1969); C. Fronsdal, Nuov. Cimento Suppl. 9, 416(1958); P.A. Moldauer and K. Case, Phys. Rev. 102, 279(1956).