Neutrino Mass and Grand Unification

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Abstract

Seesaw mechanism appears to be the simplest and most appealing way to understand small neutrino masses observed in recent experiments. It introduces three right handed neutrinos with heavy masses to the standard model, with at least one mass required by data to be close to the scale of conventional grand unified theories. This may be a hint that the new physics scale implied by neutrino masses and grand unification of forces are one and the same. Taking this point of view seriously, I explore different ways to resolve the puzzle of large neutrino mixings in grand unified theories such as SO(10) and models based on its subgroup $SU(2)_L \times SU(2)_R \times SU(4)_c$.

I. INTRODUCTION

The discovery of neutrino masses and mixings has been an important milestone in the history of particle physics and rightly qualifies as the first evidence for new physics beyond the standard model. The amount of new information on neutrinos already established from various oscillation experiments has provided very strong clues to new symmetries of particles and forces and new directions for unification. Enough puzzles have emerged making this field a hotbed for research with implications ranging from ideas such as supersymmetry and grand unification to cosmology and astrophysics.

A major cornerstone for the theory research in this field has been the seesaw mechanism introduced in the late seventies [1] to understand why neutrino masses are so much smaller than the masses of other fermions of the standard model. Even though there was no evidence for neutrino masses then, there were very well motivated extensions of the standard models that led to nonzero masses for neutrinos. It was therefore incumbent on those models that they have a mechanism for understanding why upper limits on neutrino masses known at that time were so small. Seesaw mechanism introduces three right handed neutrinos into the standard model with very large Majorana masses and predicts that observed neutrinos are their own anti-particles. A very appealing aspect of this mechanism is not only the beauty and elegance it brings to the standard model by restoring quark-lepton symmetry but also the new insight it provides into such questions as the origin of parity violation and Dirac vrs Majorana nature of the neutrino.

The first conclusive evidence for nonzero neutrino masses appeared in 1998. During the past six years, we have learnt that neutrinos not only have mass but they also mix among themselves with a pattern which is very different from that among quarks. The equation below summarizes our present knowledge about neutrino masses and mixings [2] in the notation $|\nu_\alpha>=\sum U_{\alpha i}|\nu_i>$ (where $\alpha = e, \mu.\tau$ is the flavor index and $i = 1, 2, 3$ denotes the mass eigenstate index). For the CP conserving case $U_{\alpha i}$ are functions of three angles, θ_{ij} and for these angles we have:

$$\begin{aligned} ain^2 2\theta_A \equiv sin^2 2\theta_{23} \geq 0.89 \\ \Delta m^2_A \simeq 1.4\times 10^{-3}\ eV^2 - 3.3\times 10^{-3}\ eV^2 \\ sin^2\theta_\odot \equiv sin^2\theta_{12} \simeq 0.23 - 0.37 \\ \Delta m^2_\odot \simeq 7.3\times 10^{-5}\ eV^2 - 9.1\times 10^{-5}\ eV^2 \\ sin^2\theta_{13} \leq 0.047 \end{aligned} \tag{1}$$

For the sake of comparision, note the corresponding quark mixing angles i.e. $\theta^q_{12} \simeq 0.22$; $\theta^q_{23} \simeq 0.04$ and $\theta^q_{13} \simeq 0.004$. Clearly, the mixing pattern in the lepton sector is very different from that among quarks.

It is also important to point that while the mass differences among neutrinos are fairly well determined, the situation with respect to absolute values of masses is far from clear. This is another major gap in our understanding of neutrinos compared to quarks. At present, there are three equally viable mass arrangements among the neutrinos:

- (i) Normal hierarchy i.e. $m_1 \ll m_2 \ll m_3$. In this case, we can deduce the value of $m_3 \simeq \sqrt{\Delta m^2_{23}} \equiv \sqrt{\Delta m^2_A} \simeq 0.03 - 0.07$ eV. In this case $\Delta m^2_{23} \equiv m^2_3 - m^2_2 > 0$.

The solar neutrino oscillation involves the two lighter levels. The mass of the lightest neutrino is unconstrained. If $m_1 \ll m_2$, then we get the value of $m_2 \simeq\simeq 0.008$ eV.

- (ii) Inverted hierarchy i.e. $m_1 \simeq m_2 \gg m_3$ with $m_{1,2} \simeq \sqrt{\Delta m^2_{23}} \simeq 0.03 - 0.07$ eV. In this case, solar neutrino oscillation takes place between the heavier levels and we have $\Delta m^2_{23} \equiv m^2_3 - m^2_2 < 0$.

- (iii) Degenerate neutrinos i.e. $m_1 \simeq m_2 \simeq m_3$.

There are a large number of experiments in the planning stage to improve our knowledge of mixings, to determine the mass ordering and also to find out whether neutrinos are Majorana (i.e. their own antiparticles) or Dirac fermions. These are not only crucial pieces of information about the neutrinos that we need to know to elevate our knowledge of them to the same level as the quarks but it is becoming increasingly clear that they will also point very clearly to the direction of new physics beyond the standard model. For instance if neutrinos are established to be Dirac fermions, seesaw mechanism in its simplest form will not be able to describe their masses and a major theoretical idea will be disproved.

If we accept the seesaw mechanism as the explanation for the smallness of neutrino masses, the next major challenge for theory is to understand the unusual mixing pattern among them. The hope is that in the process of understanding the mixings we will find out which of the mass patterns is realized in Nature and more importantly, will get a definite clue to the nature of new physics.

In this talk I will give some promising possibilities for this new physics and discuss their experimental tests. In particular, I will argue that the seesaw mechanism for small neutrino masses requires a scale of new physics close to the traditional scale of grand unification where all forces and matter are supposed to become unified and a new symmetry B-L which naturally arises if the gauge group is assumed to be SO(10) [5]. I will then show that a minimal version of supersymmetric SO(10) provides a very natural way to understand the large solar as well as atmospheric neutrino mixing angles while predicting a value for the mixing angle $\theta_{13} \sim 0.1 - 0.18$ depending on details. This prediction can be tested by the various planned reactor [3] and long baseline experiments [4]. I will also discuss two other related ideas which are outside the SO(10) framework but are based on one of the maximal subgroups of SO(10) i.e. $SU(2)_L \times SU(2)_R \times SU(4)_c$ [6] that also unifies quarks and leptons and then argue that measurement of the parameter θ_{13} may provide crucial insight into the question of whether there is quark-lepton unification at high scale.

While in this talk I will assume that there are only three neurtinos, we do not know for sure how many neutrinos there are. In particular if the LSND results are confirmed by the Mini Boone experiment [7], we will have evidence that there are more neutrinos and the discussions presented here will have to be extended.

II. SEESAW MECHANISM, B-L AND LEFT-RIGHT SYMMETRY

In order to introduce the seesaw mechanism, which will form the anchor for the main body of the talk, let us start with a discussion of neutrino mass in the standard model. It is based on the gauge group $SU(3)_c \times SU(2)_L \times U(1)_Y$ group under which the quarks and leptons transform as follows: Quarks: $Q_L(3, 2, \frac{1}{3})$; $u_R(3, 1, \frac{4}{3})$; $d_R(3, 1, -\frac{2}{3})$; Leptons

$L(1,2-1)$; $e_R(1,1,-2)$; Higgs Boson $\mathbf{H}(1,2,+1)$; Gluons $G_a(8,1,0)$ and Weak Gauge Fields $W^{\pm}, Z, \gamma$. The electroweak symmetry $SU(2)_L \times U(1)_Y$ is broken by the vacuum expectation of the Higgs doublet $< H^0 >= v_{wk} \simeq 180$ GeV, which gives mass to the gauge bosons and all fermions except the neutrino. The model had been a complete success in describing all known low energy phenomena, until the evidence for neutrino masses appeared.

Note that there is no right handed neutrino in the standard model and this directly leads to massless neutrinos at the tree level. The situation remains the same not only to all orders in perturbation theory but also when nonperturbative effects are taken into account. This is due to existence of an exact B-L symmetry in the theory and the absence of the right handed neutrino, N_R. The absence of the right handed neutrino from the standard model of course destroys the symmetry between quarks and leptons that is so obvious in weak interactions.

Once the right handed neutrinos (N_R) are included in the standard model, new Yukawa couplings of the form $h_\nu \bar{L} H N_R$ are allowed which after electroweak symmetry breaking lead to a neutrino mass, $M_D \equiv h_\nu v_{wk}$. Since h_ν is expected to be of same order as the charged fermion Yukawa couplings in the model, these masses are much too large to describe neutrino oscillations. Luckily, since the N_R's are singlets under the standard model gauge group, they are allowed to have Majorana masses unlike the charged fermions. We denote them by $M_R N_R^T C^{-1} N_R$ (where C is the Dirac charge conjugation matrix). The masses M_R are not constrained by the gauge symmetry and can therefore be arbitrarily large (i.e. $M_R \gg h_\nu v_{wk}$). This together with mass induced by Yukawa couplings (called the Dirac mass) leads to a the mass matrix for the neutrinos (left and right handed neutrinos together) which has the form

$$\mathcal{M}_\nu = \begin{pmatrix} 0 & M_D \\ M_D^T & M_R \end{pmatrix} \tag{2}$$

where M_D and M_R are 3×3 matrices. Diagonalizing this mass matrix, one gets the mass matrix for the light neutrinos (the seesaw formula) as:

$$\mathcal{M}_\nu = -M_D^T M_R^{-1} M_D \tag{3}$$

Since as already noted M_R can be much larger than M_D, one finds that $m_\nu \ll m_{e,u,d}$ very naturally.

Seesaw mechanism of course raises its own questions:

- Is there a natural reason for the existence of the right handed neutrinos other than quark-lepton symmetry ?
- What determines the scale of M_R ?
- Is the seesaw mechanism by itself enough to explain all aspects of neutrino masses and mixings ?

Below, we try to answer some of these questions. Restoration of quark-lepton symmetry and unification of quarks and leptons within a single gauge theory framework provided the first inspiration to bring the right handed neutrino into particle physics [6]. It is easy to see that in the presence of the N_R's, the minimal anomaly free gauge group of weak interactions expands beyond the standard model and becomes the left-right symmetric group

$SU(2)_L \times SU(2)_R \times U(1)_{B-L}$ [8] which is a subgroup of the $SU(2)_L \times SU(2)_R \times SU(4)_c$ group. This makes the weak interactions parity conserving at short distances [8], providing another appealing feature of adding the right handed neutrino. To see this explicitly, we give in Table I, the assignment of fermions and Higgs fields to the left-right gauge group.

Table I

Fields	$\mathrm{SU(2)}_L \times \mathrm{SU(2)}_R \times \mathrm{U(1)}_{B-L}$ representation
$Q_L \equiv \begin{pmatrix} u_L \\ d_L \end{pmatrix}$	$(2,1,+\frac{1}{3})$
$Q_R \equiv \begin{pmatrix} u_R \\ d_R \end{pmatrix}$	$(1,2,\frac{1}{3})$
$L_L \equiv \begin{pmatrix} \nu_L \\ e_L \end{pmatrix}$	$(2,1,-1)$
$L_R \equiv \begin{pmatrix} \nu_R \\ e_R \end{pmatrix}$	$(1,2,-1)$
ϕ	$(2,2,0)$
Δ_L	$(3,1,+2)$
Δ_R	$(1,3,+2)$

It is clear that this theory leads to a weak interaction Lagrangian of the form

$$\mathcal{L}_{wk} = \frac{g}{2}\left(\vec{j}^{\mu}_L \cdot \vec{W}_{L,\mu} + \vec{j}^{\mu}_R \cdot \vec{W}_{R,\mu}\right) \quad (4)$$

which is parity conserving prior to symmetry breaking. Furthermore, the electric charge formula is given by [9]:

$$Q = I_{3L} + I_{3R} + \frac{B-L}{2}. \quad (5)$$

where all the terms have physical meaning unlike the case of the standard model.

The left-right symmetric theories face two challenges: (i) how does the predominantly V-A nature of weak interactions emerge in such a theory and (ii) how does one understand the small neutrino masses since $SU(2)_R$ makes both the electron and the neutrino much more similar than they were in the standard model. We will see that both these challenges are met in one stroke i.e. breakdown of $SU(2)_R \times U(1)_{B-L}$ symmetry to $U(1)_Y$ not only explains the V-A nature of weak interactions but it also explain why $m_\nu \ll m_e$ via the seesaw mechanism. The seesaw scale then becomes the scale of parity violation. Furthermore, when the gauge symmetry $SU(2)_R \times U(1)_{B-L}$ is broken down while keeping the standard model symmetry unbroken, one finds from Eq. (5) the relation $\Delta I_{3R} = -\Delta \frac{B-L}{2}$. This connects $B-L$ breaking to the breakdown of parity symmetry i.e. $\Delta I_{3R} \neq 0$ and clearly implies that neutrinos must be Majorana particles.

To see this explicitly, we break the gauge symmetry of the left-right model in two stages : in stage I, vacuum expectation values (vev) of the Higgs multiplets $\Delta_R(1,3,2)$ breaks the left-right gauge symmetry to the standard model gauge group and in stage II by the bidoublet $\phi(2,2,0)$ vev breaks the standard model group to $SU(3)_c \times U(1)_{em}$. In the first stage of symmetry breaking, the right handed neutrino picks up a mass of order $f < \Delta^0_R > \equiv f v_R$.

Denoting the left and right handed neutrino by (ν, N) (in a two component notation), the mass matrix for neutrinos at this stage looks like

$$\mathcal{M}^0_\nu \;=\; \begin{pmatrix} 0 & 0 \\ 0 & fv_R \end{pmatrix} \tag{6}$$

At this stage, familiar standard model particles remain massless. As soon as the standard model symmetry is broken by the bidoublet ϕ i.e. $<\phi> \equiv \begin{pmatrix} \kappa & 0 \\ 0 & \kappa' \end{pmatrix}$, the W and Z boson as well as the fermions pick up mass. I will generically denote κ, κ' by a common symbol v_{wk}. The contribution to neutrino mass at this stage look like

$$\mathcal{M}^0_\nu \;=\; \begin{pmatrix} fv_L & hv_{wk} \\ hv_{wk} & fv_R \end{pmatrix} \tag{7}$$

Note the appearance of a new term in the neutrino mass matrix i.e. $v_L = \frac{v^2_{wk}}{v_R}$ compared to the seesaw matrix given in Eq. (1). This is a reflection of parity invariance of the model. Diagonalizing this matrix, we get a modified seesaw formula for the light neutrino mass matrix

$$\mathcal{M}_\nu \;=\; fv_L - h^T_\nu f^{-1}_R h_\nu \left(\frac{v^2_{wk}}{v_R}\right) \tag{8}$$

The important point to note is that v_L is suppressed by the same factor as the second term so that despite the new contribution to neutrino masses, seesaw suppression remains [10]. This is called the type II seesaw in contrast with the formula in Eq. (2) which is called type I seesaw formula.

An important physical meaning of the seesaw formula is brought out when it is viewed in the context of left-right models. Note that $m_\nu \to 0$ when v_R goes to infinity. In the same limit the weak interactions become pure V-A type. Therefore, left-right model derivation of the seesaw formula smoothly connects smallness of neutrino mass with suppression of V+A part of the weak interactions providing an important clarification of a major puzzle of the standard model i.e. why are weak interactions are near maximally parity violating ? The answer is that they are near maximally parity violating because the neutrino mass happens to be small.

In a subsequent section, we will discuss the connection of the seesaw mass scale with the scale of grand unification. SO(10) is the simplest gauge group that contains the right handed neutrino needed to implement the seesaw mechanism and also it is important to note that the left-right symmetric gauge group is a subgroup of the SO(10) group, which therefore provides an attractive over all grand unified framework for the discussion of neutrino masses. The extra bonus one may expect is that since bigger symmetries tend to relate different parameters of a theory, one may be able to predict neutrino masses and mixings. We will present a model where indeed this happens.

Before proceeding further, it is important to point out that type I and type II seesaw can be tested by the nature of neutrino spectrum in a model independent way. Since type I seesaw involves the Dirac mass of the neutrino, a general expectation is that it scales with generation the same way as the charged fermions of the standard model. In this case, unless there is extreme hierarchy among the right handed neutrinos, one would expect the spectrum to be hierarchical. On the other hand, it has been realized for a long time [12] that if neutrino masses are quasi-degenerate, it is a tell-tale sign of type II seesaw with the triplet vev term being the dominant one. However, a normal hierarchy can also arise with type II seesaw as we discuss in the example below. Therefore, whereas a normal hierarchy cannot distinguish between type I and type II seesaw, a quasi-degenerate spectrum is a definite sign of type II kind.

III. SEESAW AND LARGE NEUTRINO MIXINGS

While seesaw mechanism provides a simple framework for understanding the smallness of neutrino masses, it does not throw any light on the question of why neutrino mixings are large. The point is that mixings are a consequence of the structure of the light neutrino mass matrix and the seesaw mechanism is only statement about the scale of new physics. This can also be understood by doing a simple parameter counting. If we work in a basis where the right handed neutrino masses are diagonal, there are 18 parameters describing the seesaw formula for neutrino masses - three RH neutrino masses and 15 parameters in the Dirac mass matrix. On the other hand, there are only nine observables (three masses, three mixing angle and three phases) describing low energy neutrino sector. Thus there are twice as many parameters as observables. As a result, understanding neutrino mixings needs inputs beyond the simple seesaw mechanism to fix the neutrino mass matrix. Nonetheless, since the large mixings could arise from the physics involving the seesaw formula e.g. flavor structure of M_R, the large mixings are not in obvious contradiction with quark lepton unification. This becomes clear in the examples given below.

Many seesaw models for large mixings have been considered in the literature [11]. In the following section, I will focus on a recently discussed minimal SO(10) model, where without any assumption other than SO(10) grand unification, one can indeed predict all but one neutrino parameters. I will then consider a case where assumption of quasi-degeneracy in the neutrino spectrum at high scale leads in a natural way via radiative corrections to large mixings at low energies as well as briefly describe a model of quark-lepton complementarity.

To understand the fundamental physics behind neutrino mixings, we first write down the neutrino mass matrix that leads to maximal solar and atmospheric mixing for the case of normal hierarchy:

$$\mathcal{M}_\nu \;=\; \frac{\sqrt{\Delta m^2_A}}{2}\begin{pmatrix} c\epsilon & b\epsilon & d\epsilon \\ b\epsilon & 1+a\epsilon & -1 \\ d\epsilon & -1 & 1+\epsilon \end{pmatrix} \tag{9}$$

where $\epsilon \simeq \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$ and parameters a, b, c, d are of order one. Any theory of neutrino which attempts to explain the observed mixing pattern for the case of normal hierarchy must strive to get a mass matrix of this form.

It is important to point out that the above mass matrix when $a = 1$ and $b = d$, becomes symmetric under the interchange of μ and τ and yields $\theta_{13} = 0$. It was shown in two recent papers that [13], if $\mu - \tau$ symmetry is broken via $a \neq 1$ with $b = d$, then typically $\theta_{13} \sim \frac{\Delta m^2_\odot}{\Delta m^2_A}$ whereas if we have $b \neq d$, one gets $\theta_{13} \sim \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$. It turns out that most grand unified (or quark-lepton unified) theories lead to $\theta_{13} \sim \sqrt{\frac{\Delta m^2_\odot}{\Delta m^2_A}}$ (see examples below). Therefore, measurement of the mixing parameter θ_{13} may provide a way to test for possible quark-lepton unification at high scales.

IV. A PREDICTIVE MINIMAL SO(10) THEORY FOR NEUTRINOS

The main reason for considering SO(10) for neutrino masses is that its **16** dimensional spinor representation consists of all fifteen standard model fermions plus the right handed neutrino arranged according to the it $SU(2)_L \times SU(2)_R \times SU(4)_c$ [6] subgroup as follows:

$$\mathbf{\Psi} = \begin{pmatrix} u_1 & u_2 & u_3 & \nu \\ d_1 & d_2 & d_3 & e \end{pmatrix} \tag{10}$$

There are three such spinors for three fermion families.

In order to implement the seesaw mechanism in the SO(10) model, one must break the B-L symmetry. In supersymmetric SO(10) models, how B-L breaks has profound consequences for low energy physics. For instance, if B-L is broken by a Higgs field belonging to the **16** dimensional Higgs field (to be denoted by Ψ_H), then the field that acquires a nonzero vev has the quantum numbers of the ν_R field i.e. B-L breaks by one unit. In this case higher dimensional operators of the form $\Psi\Psi\Psi\Psi_H$ will lead to R-parity violating operators in the effective low energy MSSM theory such as $QLd^c, u^cd^cd^c$ etc which can lead to large breaking of lepton and baryon number symmetry and hence unacceptable rates for proton decay. This theory also has no dark matter candidate.

On the other hand, if one breaks B-L by a **126** dimensional Higgs field, the member of this multiplet that acquires vev has $B-L=2$. R-parity is therefore left as an automatic symmetry of the low energy Lagrangian. There is a naturally stable dark matter in this case. It has recently been shown that this class of models lead to a very predictive scenario for neutrino mixings [14–17]. We summarize this model below.

As already noted earlier, any theory with asymptotic parity symmetry leads to type II seesaw formula and if B-L is broken by a **126** field, then the first term in the type II seesaw formula can in principle dominate in the seesaw formula. We will discuss a model of this type below.

The basic ingredients of this model are that one considers only two Higgs multiplets that contribute to fermion masses i.e. one **10** and one **126**. A unique property of the **126** multiplet is that it not only breaks the B-L symmetry and therefore contributes to right handed neutrino masses, but it also contributes to charged fermion masses by virtue of the fact that it contains MSSM doublets which mix with those from the **10** dimensional multiplets and survive down to the MSSM scale. This leads to a tremendous reduction of the number of arbitrary parameters, as we will see below.

There are only two Yukawa coupling matrices in this model: (i) h for the **10** Higgs and (ii) f for the **126** Higgs. SO(10) has the property that the Yukawa couplings involving the **10** and **126** Higgs representations are symmetric. Therefore if we assume that CP violation arises from other sectors of the theory (e.g. squark masses) and work in a basis where one of these two sets of Yukawa coupling matrices is diagonal, then it will have only nine coupling parameters. Noting the fact that the (2,2,15) submultiplet of **126** has a pair of standard model doublets that contributes to charged fermion masses, one can write the quark and lepton mass matrices as follows [14]:

$$\begin{aligned} M_u &= h\kappa_u + f v_u \\ M_d &= h\kappa_d + f v_d \\ M_\ell &= h\kappa_d - 3 f v_d \\ M_{\nu_D} &= h\kappa_u - 3 f v_u \end{aligned} \tag{11}$$

where $\kappa_{u,d}$ are the vev's of the up and down standard model type Higgs fields in the **10** multiplet and $v_{u,d}$ are the corresponding vevs for the same doublets in **126**. The vevs added to the Yukawa couplings give a total of 13 parameters in the theory. They are determined by 13 inputs (six quark masses, three lepton masses and three quark mixing angles and weak scale). There is therefore no free parameter in the neutrino sector except for an overall seesaw scale.

To determine the light neutrino masses, we use the seesaw formula in Eq. (7), where the **f** is nothing but the **126** Yukawa coupling. These models were extensively discussed in the last decade [15] using type I seesaw formula. It was pointed out in Ref. [16] that if the direct triplet term in type II seesaw dominates, then it provides a very natural understanding of the large atmospheric mixing angle for the case of two generations without invoking any symmetries. Subsequently it was shown in Ref. [17] that the same $b-\tau$ mass convergence also provides an explanation of large solar mixing as well as small θ_{13} making the model realistic and experimentally interesting.

A simple way to see how large mixings arise in this model is to note that when the triplet term dominates the seesaw formula, we have the neutrino mass matrix $M_\nu \propto f$, where f matrix is the **126** coupling to fermions discussed earlier. Using the above equations, one can derive the following sumrule :

$$M_\nu = c(M_d - M_\ell) \tag{12}$$

To see how this leads to large atmospheric and solar mixing, let us work in the basis where the down quark mass matrix is diagonal. All the quark mixing effects are then in the up quark mass matrix i.e. $M_u = U^T_{CKM} M^d_u U_{CKM}$. Note further that the minimality of the Higgs content leads to the following sumrule among the mass matrices:

$$k\tilde{M}_\ell = r\tilde{M}_d + \tilde{M}_u \tag{13}$$

where the tilde denotes the fact that we have made the mass matrices dimensionless by dividing them by the heaviest mass of the species i.e. up quark mass matrix by m_t, down quark mass matrix by m_b etc. k, r are functions of the symmetry breaking parameters of the model. Using the hierarchical pattern of quark mixings, we can conclude that that we have

$$M_{d,\ell} \approx m_{b,\tau} \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & 1 \end{pmatrix} \tag{14}$$

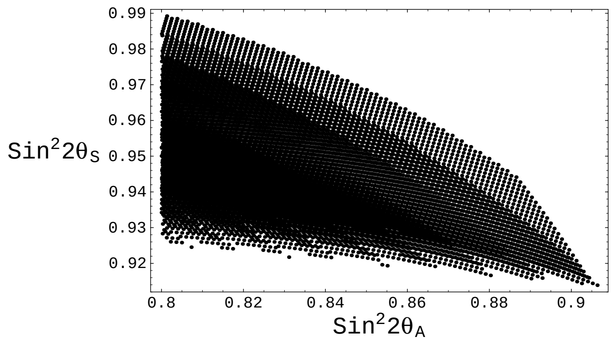

FIG. 1. The figure shows the predictions of the minimal SO(10) model for $sin^2 2\theta_\odot$ and $sin^2 2\theta_A$ for the presently range of quark masses. Note that $sin^2 2\theta_\odot \geq 0.9$ and $sin^2 2\theta_A \leq 0.9$

where $\lambda \sim 0.22$ and the matrix elements are supposed to give only the approximate order of magnitude. An important consequence of the relation between the charged lepton and the quark mass matrices in Eq. (12) is that the charged lepton contribution to the neutrino mixing matrix i.e. $U_\ell \simeq \mathbf{1}+O(\lambda)$ or close to identity matrix. As a result the neutrino mixing matrix is given by $U_{PMNS} = U_\ell^\dagger U_\nu \simeq U_\nu$. Thus the dominant contribution to large mixings will come from U_ν, which in turn will be dictated by the sum rule in Eq. (11).

To show that U_ν has two large mixings, we extrapolate the quark masses to the GUT scale and use the well known fact that $m_b - m_\tau \approx m_\tau \lambda^2$ for a wide range of values of tanβ. Using this the neutrino mass matrix $M_\nu = c(M_d - M_\ell)$ roughly takes the form

$$M_\nu = c(M_d - M_\ell) \approx m_0 \begin{pmatrix} \lambda^3 & \lambda^3 & \lambda^3 \\ \lambda^3 & \lambda^2 & \lambda^2 \\ \lambda^3 & \lambda^2 & \lambda^2 \end{pmatrix} \tag{15}$$

This mass matrix is in the form discussed in Eq. (8) and it is easy to see that both the θ_{12} (solar angle) and θ_{23} (the atmospheric angle) are now large. The detailed magnitudes of these angles of course depend on the details of the quark masses at the GUT scale. Using the extrapolated values of the quark masses and mixing angles to the GUT scale, the predictions of this model for various oscillation parameters are given in Fig. 1,2 and 3 in a self expalanatory notation. The predictions for the solar and atmospheric mixing angles fall within 3 σ range of the present central values. Note specifically the prediction in Fig. 3 for $U_{e3} \simeq 0.18$ which can be tested in MINOS as well as other planned Long Base Line neutrino experiments such as Numi-Off-Axis, JPARC etc. This model has been the subject of many investigations, which we do not discuss here [18].

A. CP violation in the minimal SO(10) model

In the discussion given above, it was assumed that CP violation is non-CKM type and resides in the soft SUSY breaking terms of the Lagrangian. The overwhelming evidence from experiments seem to be that CP violation is perhaps is of CKM type. It has recently been pointed out that with slight modification, one can include CKM CP violation in the model

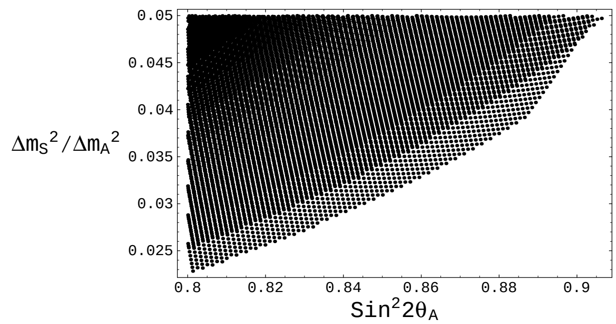

FIG. 2. The figure shows the predictions of the minimal SO(10) model for $sin^2 2\theta_A$ and $\Delta m_\odot^2/\Delta m_A^2$ for the range of quark masses and mixings that fit charged lepton masses.

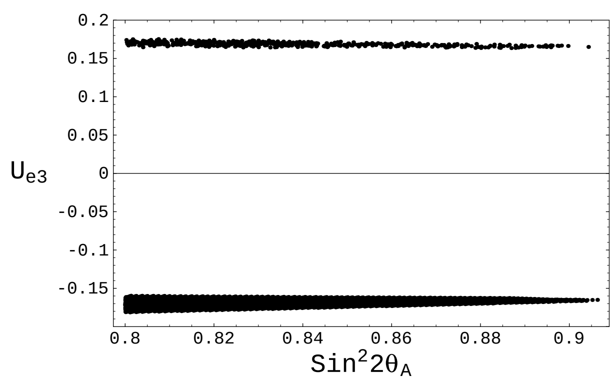

FIG. 3. The figure shows the predictions of the minimal SO(10) model for $sin^2 2\theta_A$ and U_{e3} for the allowed range of parameters in the model. Note that U_{e3} is very close to the upper limit allowed by the existing reactor experiments.

[19]. The basic idea is to include all higher dimensional operators of type $h'\Psi\Psi\bar{\Delta}\Sigma/M$ where $\bar{\Delta}$ and Σ denote respectively the **126** and the **210** dimensional representation. It is then clear that those operators transforming as **10** and **126** representations will simply redefine the h, f coupling matrices and add no new physics. On the other hand the higher dimensional operator that transforms like an effective **120** representation will add a new piece to all fermion masses. Now suppose we introduce a parity symmetry into the theory which transforms Ψ to Ψ^{c*}, then it turns out that the couplings h and f become real and symmetric matrices whereas the **120** coupling (denoted by h') becomes imaginary and antisymmetric. This process introduces three new parameters into the theory and the charged fermion masses are related to the fundamental couplings in the theory as follows:

$$\begin{aligned} M_u &= h\kappa_u + fv_u + h'v_u \qquad (16) \\ M_d &= h\kappa_d + fv_d + h'v_d \\ M_\ell &= h\kappa_d - 3fv_d - 3h'v_d \\ M_{\nu_D} &= h\kappa_u - 3fv_u - 3h'v_u \end{aligned} \qquad (17)$$

Note that the extra contribution compared to Eq. (10) is antisymmetric which therefore does not interfere with the mechanism that lead to $\mathcal{M}_{\nu,33}$ becoming small as a result of $b-\tau$ convergence. Hence the natural way that θ_A became large in the CP conserving case remains.

Let us discuss if the new model is still predictive in the neutrino sector. Of the three new parameters, one is determined by the CP violating quark phase. the two others are determined by the solar mixing angle and the solar mass difference squared. Therefore we lose the prediction for these parameters. However, we can predict in addition to θ_A which is close to maximal, $\theta_{13} \geq 0.1$ and the Dirac phase for the neutrinos. We show the predictions for Dirac phase in Fig. 4. This is a unique property of the model that it can predict the leptonic CP phase.

V. RADIATIVE GENERATION OF LARGE MIXINGS: ANOTHER APPLICATION OF TYPE II SEESAW

As alluded before, type II seesaw liberates the neutrinos from obeying normal generational hierarchy and instead could easily be quasi-degenerate in mass. This raises a new way to understand the large mixings instead of having to generate them in the original seesaw theory as is normally done. The basic idea is that at the seesaw scale, all mixings angles are small. Since the observed neutrino mixings are the weak scale observables, one must extrapolate [20] the seesaw scale mass matrices to the weak scale and recalculate the mixing angles. The extrapolation formula is

$$M_\nu(M_Z) = \mathbf{I} M_\nu(v_R) \mathbf{I} \qquad (18)$$

$$where \qquad \mathbf{I}_{\alpha\alpha} = \left(1 - \frac{h_\alpha^2}{16\pi^2}\right) \qquad (19)$$

Note that since $h_\alpha = \sqrt{2} m_\alpha / v_{wk}$ (α being the charged lepton index), in the extrapolation only the τ-lepton makes a difference. In the MSSM, this increases the $M_{\tau\tau}$ entry of the neutrino mass matrix and essentially leaves the others unchanged. It was shown in ref. [21] that if the muon and the tau neutrinos are nearly degenerate but not degenerate enough in mass at the seesaw scale, the radiative corrections can become large enough so that at the weak scale the two diagonal elements of M_ν become much more degenerate. This leads to an enhancement of the mixing angle to become almost maximal value. This can also be seen from the renormalization group equations when they are written in the mass basis [22]. Denoting the mixing angles as θ_{ij} where i, j stand for generations, the equations are:

$$\frac{ds_{23}}{dt} = -F_\tau {c_{23}}^2 \left(-s_{12}U_{\tau 1}D_{31} + c_{12}U_{\tau 2}D_{32}\right), \tag{20}$$

$$\frac{ds_{13}}{dt} = -F_\tau c_{23}{c_{13}}^2 \left(c_{12}U_{\tau 1}D_{31} + s_{12}U_{\tau 2}D_{32}\right), \tag{21}$$

$$\begin{aligned}\frac{ds_{12}}{dt} = & -F_\tau c_{12} \left(c_{23}s_{13}s_{12}U_{\tau 1}D_{31} - c_{23}s_{13}c_{12}U_{\tau 2}D_{32}\right. \\ & \left.+U_{\tau 1}U_{\tau 2}D_{21}\right).\end{aligned} \tag{22}$$

where $D_{ij} = (m_i + m_j)) / (m_i - m_j)$ and $U_{\tau 1,2,3}$ are functions of the neutrino mixings angles. The presence of $(m_i - m_j)$ in the denominator makes it clear that as $m_i \simeq m_j$, that particular coefficient becomes large and as we extrapolate from the GUT scale to the weak scale, small mixing angles at GUT scale become large at the weak scale. It has been shown recently that indeed such a mechanism for understanding large mixings can work for three generations [23]. It was shown that if we identify the seesaw scale neutrino mixing angles with the corresponding quark mixings and assume quasi-degenerate neutrinos, the weak scale solar and atmospheric angles get magnified to the desired level while due to the extreme smallness of V_{ub}, the magnified value of U_{e3} remains within its present upper limit. In figure 5, we show the evolution of the mixing angles to the weak scale. A requirement for this scenario to work is that the common mass of neutrinos must be larger than 0.1 eV, a result that can be tested in neutrinoless double beta experiments.

VI. QUARK-LEPTON COMPLEMENTARITY AND LARGE SOLAR MIXING

There has been a recent suggestion [24] that perhaps the large but not maximal solar mixing angle is related to physics of the quark sector. According to this, the deviation from maximality of the solar mixing may be related to the quark mixing angle $\theta_C \equiv \theta_{12}^q$ and is based on the observation that the mixing angle responsible for solar neutrino oscillations, $\theta_\odot \equiv \theta_{12}^\nu$ satisfies an interesting complementarity relation with the corresponding angle in the quark sector $\theta_{Cabibbo} \equiv \theta_{12}^q$ i.e. $\theta_{12}^\nu + \theta_{12}^q \simeq \pi/4$. While it is quite possible that this relation is purely accidental or due to some other dynamical effects, it is interesting to pursue the possibility that there is a deep meaning behind it and see where it leads. It has been shown in a recent paper that if Nature is quark lepton unified at high scale, then a relation between θ_{12}^ν and θ_{12}^q can be obtained in a natural manner provided the neutrinos obey the inverse hierarchy [25]. It predicts $sin^2\theta_\odot \simeq 0.34$ which agrees with present data at the 2σ level. It also predicts a large $\theta_{13} \sim 0.18$, both of which are predictions that can be tested experimentally in the near future.

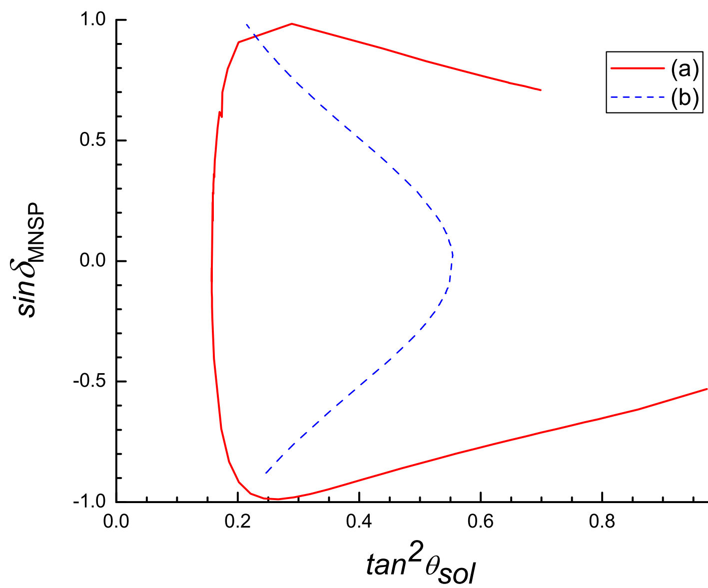

FIG. 4. The prediction of MNSP phase is plotted as a function of the solar mixing angle. The two lines (a) and (b) correspond to two choices of signs of the fermion masses. The phases are plotted for $\Delta m^2_{sol}/\Delta m^2_A = 0.02$.

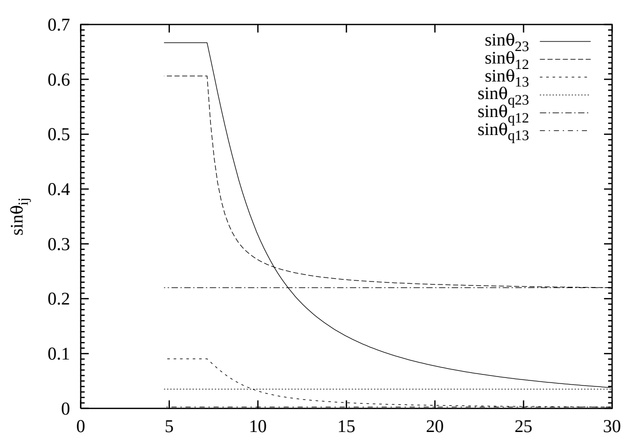

FIG. 5. Radiative magnification of small quark-like neutrino mixings at the see-saw scale to bilarge values at low energies. The solid, dashed and dotted lines represent $\sin\theta_{23}$, $\sin\theta_{13}$, and $\sin\theta_{12}$, respectively.

VII. CONCLUSION

In summary, the seesaw mechanism is by far the simplest and most appealing way to understand neutrino masses. It not only improves the aesthetic appeal of the standard model by restoring quark-lepton symmetry but it also makes weak interactions asymptotically parity conserving. Further more it connects neutrino masses with the hypothesis of grand unification. In this talk I have discussed three ways to understand the large solar and atmospheric neutrino mixings within the frameworks that unify quarks and lepton and in one case into a grand unified model based on SO(10). All three models predict large values for θ_{13} and can therefore be tested in forthcoming experiments. The SO(10) model appears to be most promising since it not only resolves the difficulties of the minimal SUSY SU(5) GUT but is also a minimal predictive model for neutrinos.

From these examples, one is also tempted to conclude that a large θ_{13} could be a generic feature of models that unify quarks and leptons, which if true will be a unique window to a very important question in beyond the standard model physics.

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