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Electromagnetic Contributions to the Schiff Moment*

Scott Thomas

Santa Cruz Institute for Particle Physics

University of California, Santa Cruz, CA 95064

Abstract

The Schiff moment, $\bar{\psi}_i \overleftrightarrow{\partial_\mu} \gamma_5 \psi_i \, \bar{\psi}_j \gamma^\mu \psi_j$, is a parity and time reversal violating fermion-fermion coupling. The nucleus-electron Schiff moment generically gives the most important contribution to the electric dipole moments of atoms and molecules with zero net intrinsic electronic spin and nuclear spin $\frac{1}{2}$. Here, the electromagnetic contribution to the Schiff moment, $\bar{\psi}_i \overleftrightarrow{\partial_\nu} \gamma_5 \psi_i \, \partial_\mu F^{\mu\nu}$, is considered. For a nucleon, the leading chirally violating contribution to this interaction is calculable in the chiral limit in terms of the parity and time reversal violating pion-nucleon coupling. For the Schiff moment of heavy nuclei, this chiral contribution is somewhat smaller than the finite size effect discussed previously in the literature.

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Experimental searches for electric dipole moments (EDMs) provide low energy probes for P and T violating physics at and beyond the electroweak symmetry breaking scale. The current experimental limits on the EDMs of atoms, molecules, and the neutron already give bounds on certain T violating extensions of the standard model. The sensitivity of atomic and molecular EDMs to microscopic P and T violation depends on the net intrinsic electronic spin. Atoms with an unpaired electron, such as ^{133}Cs and ^{205}Tl, are sensitive mainly to the electron EDM. Atoms with paired electrons, such as ^{199}Hg and ^{129}Xe, or the molecule ^{205}TlF, are sensitive mainly to nuclear effects. The effect of the nuclear EDM is highly suppressed due to Schiff's theorem.[1] Higher electromagnetic moments are not affected by Schiff's theorem. This allows the magnetic quadrupole moment to contribute for nuclei with $J \geq 1$, where J is the nuclear spin. For $J = \frac{1}{2}$ however, the dominant nuclear contribution to the atomic or molecular EDM comes from a local (on the atomic scale) coupling between the nucleus and electrons known as the Schiff moment.

The Schiff moment (SM) coupling two spin $\frac{1}{2}$ Dirac fermions arises from the operators

$$S_1 \; \bar{\psi}_i \overleftrightarrow{\partial_\mu} \gamma_5 \psi_i \; \bar{\psi}_j \gamma^\mu \psi_j \tag{1}$$

$$-S_2 \; \bar{\psi}_i \sigma^{\mu\nu} i\gamma_5 \psi_i \; \partial_\nu (\bar{\psi}_j \gamma_\mu \psi_j) \tag{2}$$

These operators are equivalent on shell and can therefore be related using equations of motion. In what follows only (1) will be kept explicitly. The contribution of the nucleus-electron SM to an atomic EDM can be estimated just on dimensional grounds to be $d_a \sim eS\ Z^2 \alpha m_e^2$, where m_e is the electron mass.[3−5] Similarly, a molecular EDM can be estimated to be $d_m \sim eS\ Z^2 \alpha m_e m_N$, where m_N is the nucleus mass.[3−5]

The magnitude of S depends on the origin of the microscopic P and T violation and the scale at which the effective operator (1) is generated. One contribution comes from the neutral current component of the weak electric dipole moment

(WEDM), $-d_{\rm w}\ \frac{1}{2}\bar{\psi}_i\sigma^{\mu\nu}i\gamma_5\psi_i Z_{\mu\nu}$, where $Z_{\mu\nu}$ is the Z boson field strength. Such an operator would be generated, for example, at one loop in a multi-Higgs model of T violation. For light fermions the chirally violating WEDM is effectively dimension six, being suppressed by two powers of the heavy scale associated with T violation. Tree level Z exchange, with the WEDM, then gives a SM (1) with $S = g_{\rm v} d_{\rm w}/m_{\rm z}^2$, where $g_{\rm v}$ is the neutral current vector coupling of the fermion j.[2] A SM can also arise directly at the heavy scale. For example, in the supersymmetric standard model box diagrams involving gauginos and the scalar partners of the external fermions give $S \sim \sin\phi\ \alpha^2 m_i/M_{\rm SUSY}^4$, where $\sin\phi$ is some combination of T violating phases.[2] The nucleus-electron SM arising from quark-electron moments of the type discussed above are suppressed by four powers of a heavy mass. The resulting atomic or molecular EDM is therefore less important than that arising from, for example, the light quark EDM or chromo-electric dipole moment (CEDM), which are suppressed by only two powers of a heavy mass.

A SM may also arise from the electromagnetic interaction

$$S'\ \bar{\psi}_i \overleftrightarrow{\partial_\nu} \gamma_5 \psi_i\, \partial_\mu F^{\mu\nu} \tag{3}$$

Using the equation of motion $\partial_\mu F^{\mu\nu} = eQ_j\ \bar{\psi}_j\gamma^\nu\psi_j$ gives the operator (1) with $S = eQ_j S'$. Diagrammatically, (1) arises from the coupling of the electromagnetic current to (3) through tree level photon exchange. The q^2 dependence in (3) is canceled by the photon propagator. In order to make explicit the origin of the operator (3) consider the matrix element of the electromagnetic current for particle i. The most general P and T odd matrix element of j_μ on two single particle Dirac states can be written

$$\langle p', s'|j^\mu|p, s\rangle = D(q^2)\ \bar{u}(p', s')\sigma^{\mu\nu}\gamma_5 q_\nu u(p, s) \tag{4}$$

where $q_\nu = (p-p')_\nu$, and $D(q^2)$ is a momentum dependent form factor. Expanding about $q^2 = 0$, the constant part of $D(q^2)$ is just the electric dipole moment, d,

$$d = D(0) \tag{5}$$

The q^2 dependent piece of $D(q^2)$ is reproduced for on shell fermions by the operator

(3) with

$$S' = \frac{d}{dq^2} D(0) \tag{6}$$

As long as $D(q^2)$ is not constant, any process which produces an EDM also produces a SM.* In particular, a quark-electron electromagnetic SM can be generated at the heavy scale associated with T violation by the same processes responsible for a quark EDM. However, this will necessarily be suppressed by four powers of the heavy scale, just as the previous contributions.

More important are contributions to the nucleus-electron SM arising at the nuclear scale. First consider the nucleon-electron SM. In the chiral limit, effective operators involving nucleons are typically dominated by nonanalytic contributions arising from integrating out pions. As an example of the nonanalytic contribution to the nucleon SM consider the chirally violating P and T odd pion-nucleon coupling

$$\bar{g} \bar{N} \pi N \tag{7}$$

where $\pi = \tau^a \pi^a$. This coupling could arise from a finite QCD vacuum angle or a light quark CEDM.[2] The nonanalytic contribution comes from the same graphs which give a nucleon EDM (see fig. 1).[6] This graph for the SM is divergent in the infrared, cutoff by the pion mass. A straightforward calculation gives

$$S' = \frac{e \bar{g} g_A}{48\pi^2 f_\pi m_\pi^2} \tag{8}$$

where $g_A \simeq 1.26$ is the usual pion-nucleon coupling, and $f_\pi \simeq 93$ MeV is the pion decay constant. Notice that since $\bar{g}$ scales as m_π^2, S' is a constant in the chiral limit. This is in contrast to the nucleon EDM from (7) which scales as $m_\pi^2 \ln m_\pi^2$ in the chiral limit.[6] The SM of individual nucleons will contribute incoherently to the nucleus SM. This nonanalytic contribution to the SM of heavy spin $\frac{1}{2}$ nuclei will therefore be equal to the nucleon SM (8).

$\star$ It is worth noting that Eqs. (4) and (6) show that the electromagnetic contribution to the SM can be thought of as the P and T odd analog of the charge radius or electromagnetic anapole moment.

In addition to the incoherent contribution to the nucleus-electron SM, there are coherent contributions due to the finite size of the nucleus. The magnitude of the finite size effects may be estimated with a simple model due to Sushkov, Flambaum, and Khriplovich.[3−5,7] The model assumes a nucleus with a single unpaired valence proton. In the nonrelativistic limit (7) leads to a coupling of the valence proton to the nuclear core of

$$\frac{\bar{g} g_A}{f_\pi m_\pi^2}\ \vec{\sigma} \cdot \vec{\nabla} \rho \tag{9}$$

where ρ is the core density. To model the effect of this interaction the nuclear potential, U, is assumed to be proportional to ρ, i.e. $U = \rho(U_o/\rho_o)$, where $\rho_o \sim \tilde{m}^3$ and $U_o \sim \tilde{m}^3/4\pi f_\pi^2$ are the density and potential deep in the core, and $\tilde{m}$ is some nuclear mass parameter characterizing the repulsive part of the nuclear potential ($\tilde{m}$ is independent of the chiral limit). Under this assumption the interaction (9) leads to a constant shift of the valence proton wave function given by

$$\vec{\lambda} \simeq \frac{4\pi \bar{g} g_A f_\pi}{m_\pi^2}\ \vec{\sigma} \tag{10}$$

This constant shift leads to P and T odd interactions of the nucleus with the electromagnetic field. For spin $\frac{1}{2}$ these are contained in the form factor $D(q^2)$. Just on dimensional grounds the electromagnetic SM resulting from the shift $\vec{\lambda}$ is[3,7]

$$S' \sim \frac{e}{4\pi} \lambda R^2 \sim \frac{e \bar{g} g_A f_\pi A^{2/3}}{m_\pi^2 \tilde{m}^2} \tag{11}$$

where $R \sim A^{1/3} \tilde{m}^{-1}$ is the rms radius of the valence wave function, and A the atomic number. The finite size effect is essentially coherent over the entire nucleus, being proportional to the square of the valence nucleon wave function radius. It is larger than the incoherent loop contribution (8) by $\mathcal{O}((4\pi f_\pi/\tilde{m})^2 A^{2/3})$, where $(4\pi)^2$ counts the loop factor.

In conclusion, any P and T violating microscopic physics which generates EDMs also generally gives rise to an electromagnetic SM. The nucleus-electron SM represents the most important nuclear contribution to the EDMs of atoms and molecules with paired electrons and nuclear spin $\frac{1}{2}$. This moment arises predominantly from indirect effects at the nuclear scale rather than directly from the microscopic P and T violating scale. The chirally violating contribution to the nucleon SM is nonanalytic and calculable in the chiral limit. Coherent finite size effects however dominate the SM of heavy nuclei.

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FIGURE CAPTIONS

1) Nonanalytic contribution to the nucleon electromagnetic Schiff moment from the coupling $\bar{g}\bar{N}\pi N$. Other graphs related by gauge invariance are not shown. The graph with the photon attached to the nucleon is smaller by $\mathcal{O}(m_\pi^2/m_n^2)$.