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Division Algebras, (1,9)-Space-time, Matter-antimatter Mixing

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Abstract

The tensor product of the division algebras, which is a kernel for the structure of the Standard Model, is also a root for the Clifford algebra of (1,9)-space-time. A conventional Dirac Lagrangian, employing the (1,9)-Dirac operator acting on the Standard Model hyperfield, gives rise to matter into antimatter transitions not mediated by any gauge field. These transitions are eliminated by restricting the dependencies of the components of the hyperfield on the extra six dimensions, which appear in this context as a complex triple.

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This article is an extension of my work on applying the tensor product of the division algebras to the lepto-quark Standard Model [1-4] and beyond. Although it is selfcontained, many results derived previously are not rederived here.

Applications of the division algebras to particle physics [5-10] are not new, nor are all the same. This application, to the best of my knowledge, while owing a debt to the work of Gürsey and Günaydin, is the only one of its kind. Like all applications of these algebras, however, it is motivated by the attractive notion that the special structures of mathematics play a role in the design of reality. Most theorists share a faith - or at least a hope - of this sort; here it has been allowed to become a guiding principle.

In this article I present the first radical extension of my ideas beyond the Standard Model and its foundation. Because it combines the Standard Model with (1,9)-space-time ($\mathbf{R}^{1,9}$), it may well prove a step toward the development of a connection to, and a narrowing of, string theory, the initial euphoria to which has - in the fashion of GUTs and SUSY - succumbed to the curse of multiple realities.

The nontrivial real division algebras with unity are the complexes, **C**, quaternions, **Q**, and octonions, **O**. They are 2-, 4-, and 8-dimensional. Multiplication tables for **Q** and **O** are constructable from the following elegant rules:

Division Algebra	**Q**	**O**	
Imaginary Units	$q_i, 1 = 1, 2, 3,$	$e_a, a = 1, ..., 7,$	
Anti-commutators	$q_i q_j + q_j q_i = 2\delta_{ij},$	$e_a e_b + e_b e_a = 2\delta_{ab},$	(1)
Cyclic Rules	$q_i q_{i+1} = q_{i-1} = q_{i+2},$	$e_a e_{a+1} = e_{a-2} = e_{a+5},$	
Index Doubling	$q_i q_j = q_k \Longrightarrow$ $q_{(2i)} q_{(2j)} = -q_{(2k)},$	$e_a e_b = e_c \Longrightarrow$ $e_{(2a)} e_{(2b)} = e_{(2c)},$	

where **Q**-indices run from 1 to 3, modulo 3, and **O**-indices run from 1 to 7, modulo 7.

$\mathbf{C}\otimes\mathbf{Q}$ is spanned by the 8 elements $\{1, i, q_j, iq_j\}$. It is isomorphic to the Pauli algebra, $\mathbf{C}(2)$, which is the Clifford algebra of $\mathbf{R}^{3,0}$ space. Represented by $\mathbf{C}(2)$, the spinors of that Clifford algebra are 2×1 over $\mathbf{C}$, the so-called Pauli or Weyl spinors. The spinor space of $\mathbf{C}\otimes\mathbf{Q}$, however, is 1×1 over $\mathbf{C}\otimes\mathbf{Q}$, hence is $\mathbf{C}\otimes\mathbf{Q}$ itself. In this case, to distinguish the Clifford algebra from its spinor space, we denote the former $\mathbf{C}_L\otimes\mathbf{Q}_L$, the subscript indicating action from the left on the spinor space, which we denote $\mathbf{C}\otimes\mathbf{Q}$.

$\mathbf{C}\otimes\mathbf{Q}$ is twice as large as it needs to be. It is the direct sum of two 2-dimensional (over $\mathbf{C}$) Weyl spinor spaces unmixed by $\mathbf{C}_L \otimes \mathbf{Q}_L$ (just as $\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$ in $\mathbf{C}(2)$ is the direct sum of the Weyl spinor spaces $\begin{bmatrix} x_1 & 0 \\ x_2 & 0 \end{bmatrix}$ and $\begin{bmatrix} 0 & y_1 \\ 0 & y_2 \end{bmatrix}$). If $\vec{x} \in \mathbf{Q}$ satisfies $\vec{x}^2 = -1$, then multiplication from the right on $\mathbf{C}\otimes\mathbf{Q}$ by the idempotents $\frac{1}{2}(1 \pm i\vec{x})$ projects two such Weyl spinor spaces (just as multiplication from the right by the idempotents $\frac{1}{2}(\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \pm \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix})$ on $\begin{bmatrix} x_1 & y_1 \\ x_2 & y_2 \end{bmatrix}$ projects the $\mathbf{C}(2)$ Weyl spinor spaces above). $\mathbf{Q}_R$, which acts from the right on $\mathbf{C}\otimes\mathbf{Q}$, mixes these two independent spinor spaces. $\mathbf{Q}_R$ commutes with $\mathbf{C}_L \otimes \mathbf{Q}_L$, so it is an "internal" algebra, where the Clifford (geometric) algebra is "external". The elements of unit length of $\mathbf{Q}_R$ form the group $SU(2)$, which in previous work along these lines was manifested as the isospin gauge symmetry [1].

The octonion algebra is generally considered ill-suited to Clifford algebra theory because $\mathbf{O}$ is nonassociative, and Clifford algebras are associative. This problem disappears once we identify $\mathbf{O}$ as the spinor space of $\mathbf{O}_L$, the adjoint algebra of actions of $\mathbf{O}$ on itself from the left. $\mathbf{O}_L$ *is* associative. $\mathbf{O}_L$ is linear in actions of the form

$$e_{Lab...c}[x] = e_a(e_b(...(e_c x)...)), \tag{2}$$

$x \in \mathbf{O}$. For example, although $e_1 e_2 = e_6$,

$$e_{L12}[x] = e_1(e_2 x) \neq e_6 x = e_{L6}[x]$$

in general; and although $e_1(e_2 e_4) = e_7$,

$$e_{L124}[x] = e_1(e_2(e_4 x)) \neq e_7 x = e_{L7}[x]$$

in general. These are consequences of nonassociativity. The elements $e_{Lab...c}$ satisfy

$$
\begin{aligned}
e_{Labcc...d} &= -e_{Lab...d}, \\
e_{Lab...c} &= \pm e_{Lpq...r},
\end{aligned}
\tag{3}
$$

pq...r an even-odd permutation of ab...c, and

$$
e_{Lab...c}e_{Ldf...g} = e_{Lab...cdf...g}. \tag{4}
$$

It is also not difficult to prove that $e_{L7654321}[x] = x$ for all x in $\mathbf{O}$. Therefore, for example, using (4) and (5) one can easily prove

$$
e_{L4567} = e_{L4567}e_{L7654321} = e_{L321}. \tag{5}
$$

That is, any element of $\mathbf{O}_L$ with four or more indices can be reduced to an element with three indices or less. So a complete basis for $\mathbf{O}_L$ consists of the elements

$$
1,\ e_{La},\ e_{Lab},\ e_{Labc}. \tag{6}
$$

Therefore $\mathbf{O}_L$ is 1+7+21+35=64-dimensional, and $\mathbf{O}_L \simeq \mathbf{R}(8)$. The embedding of parentheses in the definition (2), implying (4), trivially implies $\mathbf{O}_L$ is associative.

$\mathbf{O}_L$ is isomorphic to the Clifford algebra of the space $\mathbf{R}^{0,6}$, the spinor space of which is 8-dimensional over $\mathbf{R}$. In this case the spinor space is $\mathbf{O}$ itself, the object space of $\mathbf{O}_L$. It is significant that the dimensionality of $\mathbf{O}$ is correct in this case. This is tied to the remarkable fact that the algebra $\mathbf{O}_R$ of right adjoint actions of $\mathbf{O}$ on itself is the *same* algebra as $\mathbf{O}_L$. Every action in $\mathbf{O}_R$ can be written as an action in $\mathbf{O}_L$.

A 1-vector basis for $\mathbf{O}_L$, playing the role of the Clifford algebra of $\mathbf{R}^{0,6}$, is $\{e_{Lp}, p = 1,...,6\}$. The resulting 2-vector basis is then $\{e_{Lpq}, p, q = 1,...,6, p \neq q\}$. This subspace is 15-dimensional, closes under the commutator product, and is in that case isomorphic to $so(6)$. The intersection of this Lie algebra with the Lie algebra of the automorphism group of $\mathbf{O}$, G_2, is $su(3)$, with a basis

$$
su(3) \rightarrow \{e_{Lpq} - e_{Lrs}, p, q, r, s \text{ distinct, and from 1 to 6}\}. \tag{7}
$$

The group $SU(3)$ generated by these elements arises as the color gauge group in applications [1] (note that $SU(3)$ is the stability group of e_7, hence the index doubling automorphism of $\mathbf{O}$ is an $SU(3)$ rotation).

Finally we let $\mathbf{C}\otimes\mathbf{Q}\otimes\mathbf{O}$ play the role of spinor space to $\mathbf{C}_L\otimes\mathbf{Q}_L\otimes\mathbf{O}_L$, which is isomorphic to $\mathbf{C}(16)$, hence isomorphic to the Clifford algebra of the space $\mathbf{R}^{0,9}$. With respect to the gauge symmetry $SU(2)\times SU(3)$ outlined above, which expands to $U(2)\times U(3)$ [1], the spinor space $\mathbf{C}\otimes\mathbf{Q}\otimes\mathbf{O}$ transforms exactly like the direct sum of a family and antifamily of leptoquark Weyl spinors. Quantum numbers for the (family) spinors can be manifested in two ways, one corresponding to righthanded particles, one to lefthanded. They can be simultaneously incorporated by expanding $\mathbf{C}_L\otimes\mathbf{Q}_L\otimes\mathbf{O}_L$ to $\mathbf{C}_L\otimes\mathbf{Q}_L\otimes\mathbf{O}_L(2)$ (2×2 over $\mathbf{C}_L\otimes\mathbf{Q}_L\otimes\mathbf{O}_L$), the "Dirac" algebra for $\mathbf{R}^{1,9}$ space-time (just as $\mathbf{C}_L\otimes\mathbf{Q}_L(2)$, isomorphic to $\mathbf{C}(4)$, is the Dirac algebra for $\mathbf{R}^{1,3}$). The spinor space in this case is 2×1 over $\mathbf{C}\otimes\mathbf{Q}\otimes\mathbf{O}$.

Let Ψ be such a spinor, and give it a functional dependence on $\mathbf{R}^{1,9}$ space-time. Let

$$\rho_{\pm} = (1\pm ie_7)/2. \tag{8}$$

Then $\rho_+\Psi$ is the matter half of Ψ, and $\rho_-\Psi$ the antimatter half. $\rho_+\Psi\rho_+$ is an $SU(2)$ lepton doublet, and $\rho_+\Psi\rho_-$ is a quark $SU(2)$ doublet, $SU(3)$ triplet (reverse signs for antimatter).

Define in $\mathbf{R}(2)$:

$$\epsilon = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \alpha = \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}, \beta = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}, \omega = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}.$$

A 1-vector basis for the Clifford algebra of $R^{1,9}$ consists of the elements:

$$\gamma_0 = \beta,\;\; \gamma_j = q_j e_{L7}\omega, j=1,2,3,\;\; \gamma_h = ie_{h-3}\omega, h=4,...,9. \tag{9}$$

These satisfy:

$$\gamma_h\gamma_l + \gamma_l\gamma_h = 2\eta_{hl}\epsilon,$$

η_{hl} diagonal $(1(+),9(-))$.

The (1,9)-Dirac operator is $\not{\partial}_{1,9} = \gamma_f\partial^f, f=0,1,...,9,$ and I define $\not{\partial}_{1,3} = \gamma_\mu\partial^\mu, \mu=0,1,2,3,$ $\not{\partial}_{0,6} = \not{\partial}_{1,9} - \not{\partial}_{1,3}$. Define

$$\rho_{L\pm} = (1\pm ie_{L7})/2 \tag{10}$$

(the left adjoint version of $\rho_\pm$). Using these adjoint idempotents we can decompose $\not{\partial}_{1,9}$ into its (1,3)- and (0,6)-Dirac operator parts, one of each for both matter and antimatter:

$$\begin{aligned}\partial\!\!\!/_{1,9} &= \rho_{L+}\partial\!\!\!/_{1,9}\rho_{L+} + \rho_{L-}\partial\!\!\!/_{1,9}\rho_{L-} + \rho_{L+}\partial\!\!\!/_{1,9}\rho_{L-} + \rho_{L-}\partial\!\!\!/_{1,9}\rho_{L+} \\ &= \rho_{L+}\partial\!\!\!/_{1,3}\rho_{L+} + \rho_{L-}\partial\!\!\!/_{1,3}\rho_{L-} + \rho_{L+}\partial\!\!\!/_{0,6}\rho_{L-} + \rho_{L-}\partial\!\!\!/_{0,6}\rho_{L+}, \\ &= \partial\!\!\!/_{1,3}\rho_{L+} + \partial\!\!\!/_{1,3}\rho_{L-} + \partial\!\!\!/_{0,6}\rho_{L-} + \partial\!\!\!/_{0,6}\rho_{L+} \end{aligned} \tag{11}$$

(note that $\partial\!\!\!/_{1,3}\rho_{L\pm}$ are the matter/antimatter Dirac operators for (1,3)-space-time, and that because $e_{L7}\rho_{L\pm} = \mp i\rho_{L\pm}$, the partials of the latter are space-reflected relative to the former). Therefore,

$$\begin{aligned}\partial\!\!\!/_{1,9}\Psi &= (\partial\!\!\!/_{1,3}\rho_{L+} + \partial\!\!\!/_{1,3}\rho_{L-} + \partial\!\!\!/_{0,6}\rho_{L-} + \partial\!\!\!/_{0,6}\rho_{L+})\Psi \\ &= \partial\!\!\!/_{1,3}(\rho_+\Psi) + \partial\!\!\!/_{1,3}(\rho_-\Psi) + \partial\!\!\!/_{0,6}(\rho_-\Psi) + \partial\!\!\!/_{0,6}(\rho_+\Psi). \end{aligned} \tag{12}$$

To form a Lagrangian for the field we use the inner product of $\mathbf{C}\otimes\mathbf{Q}\otimes\mathbf{O}$ [1]:

$$\begin{gathered}\mathcal{L} =< \Psi, \partial\!\!\!/_{1,9}\Psi > \\ =< \rho_+\Psi + \rho_-\Psi, \partial\!\!\!/_{1,3}(\rho_+\Psi) + \partial\!\!\!/_{1,3}(\rho_-\Psi) + \partial\!\!\!/_{0,6}(\rho_-\Psi) + \partial\!\!\!/_{0,6}(\rho_+\Psi) > \\ =< \rho_+\Psi, \partial\!\!\!/_{1,3}(\rho_+\Psi) > + < \rho_-\Psi, \partial\!\!\!/_{1,3}(\rho_-\Psi) > \\ + < \rho_+\Psi, \partial\!\!\!/_{0,6}(\rho_-\Psi) > + < \rho_-\Psi, \partial\!\!\!/_{0,6}(\rho_+\Psi) > \end{gathered} \tag{13}$$

(the last equality arising from the algebra of the inner product). The first two terms after the last equality in (13), $< \rho_\pm\Psi, \partial\!\!\!/_{1,3}(\rho_\pm\Psi) >$, are ordinary. One can obtain a list of viable particle transitions from such Lagrangians, as each Weyl component of Ψ has an obvious particle identification. For example, after gauging $U(2) \times U(3)$, algebraic combinations of spinor and gauge fields that survive the inner product correspond to viable transitions (this aspect won't be developed further here; see [1],[11]). These first two terms connect matter/antimatter to matter/antimatter ($\rho_\pm\Psi$= matter/antimatter), and upon gauging $U(2) \times U(3)$ give rise to an unconventional looking version of the Standard Model.

The last two terms of (13), $< \rho_\mp\Psi, \partial\!\!\!/_{0,6}(\rho_\pm\Psi) >$, are a problem, even without gauge fields, for they imply matter/antimatter ($\rho_\pm\Psi$) into antimatter/matter ($\rho_\mp\Psi$) transitions, mediated algebraically by $\partial\!\!\!/_{0,6}$. As such transitions are unobserved, the rest of the article will be devoted to getting rid of the last two terms of (13).

The 2-vector basis for the Clifford algebra of $\mathbf{R}^{1,9}$, derived from the 1-vectors in (8), is

$$q_j\epsilon,\ q_je_{L7}\alpha,\ ie_{Lp}\alpha,\ iq_je_{Lp7}\epsilon,\ e_{Lpq}\epsilon, \tag{14}$$

j=1,2,3, p,q∈ $\{1, ..., 6\}$. This 45-dimensional subspace closes under the commutator product and is in that case isomorphic to $so(1,9)$. The first six elements, $\{q_j\epsilon,\ q_j e_{L7}\alpha\}$, form a basis for $so(1,3)$, the last fifteen, $\{e_{Lpq}\epsilon\}$, a basis for $so(6)$. This is the same $so(6)$ we saw earlier, and it contains color $su(3)$ (see (7)). That is, the space $\mathbf{R}^{0,6}$, hence $\partial\!\!\!/_{0,6}$, carry color charges (one consequence of these charges: in none of the unwanted transitions implied by (13) can a particle make a transition to its own antiparticle; hence, for example, quarks may mix with anitleptons, violating baryon and lepton number conservation).

Consider the element $\partial\!\!\!/_{0,6}(\rho_+\Psi)$ which appears in the last term of (13). Because

$$\rho_\pm e_7 = \mp i\rho_\pm,\ \rho_\pm e_5 = \mp i\rho_\pm e_1,\ \rho_\pm e_3 = \mp i\rho_\pm e_2,\ \rho_\pm e_6 = \mp i\rho_\pm e_4, \qquad (15)$$

$\rho_+\Psi$ may be decomposed into

$$\rho_+\Psi = \rho_+[\Psi_+^0 + \Psi_+^1 e_1 + \Psi_+^2 e_2 + \Psi_+^4 e_4], \qquad (16)$$

where the Ψ_+^m, m=0,1,2,4, are 2×1 over $\mathbf{C}\otimes\mathbf{Q}$. These four fields can be designated lepton, red-, green-, and blue-quark.

Now consider $\partial\!\!\!/_{0,6}(\rho_+\Psi_+)$, and in particular, for example, the term (sum p=1,...,6)

$$\begin{aligned}
&\partial\!\!\!/_{0,6}(\rho_+\Psi_+^1 e_1) = i\omega e_p\partial^{p+3}[\rho_+\Psi_+^1 e_1]\\
&= i\omega(\rho_- e_1\partial^4 + \rho_+ e_2\partial^5 + \rho_+ e_3\partial^6 + \rho_+ e_4\partial^7 + \rho_- e_5\partial^8 + \rho_+ e_6\partial^9)[\Psi_+^1 e_1]\\
&= i\omega(\rho_- e_1(\partial^4 + i\partial^8) + \rho_+ e_2(\partial^5 - i\partial^6) + \rho_+ e_4(\partial^7 - i\partial^9))[\Psi_+^1 e_1]\\
&= i\omega(e_1(\partial^4 + i\partial^8) + e_2(\partial^5 - i\partial^6) + e_4(\partial^7 - i\partial^9))[\rho_+\Psi_+^1 e_1]\\
&\equiv \partial\!\!\!/_{6+--}[\rho_+\Psi_+^1 e_1] \qquad (17)
\end{aligned}$$

(in the second line the nonassociativity of $\mathbf{O}$ plays a part in altering the sign subscripts of $\rho_\pm$; in general nonassociativity plays an essential role in keeping the mathematics consistent with phenomenology). $\partial\!\!\!/_{6+--}$ (generalized below) is defined in the penultimate line. In like manner one can demonstrate that

$$\begin{aligned}
\partial\!\!\!/_6\rho_+\Psi_+^0 &= \partial\!\!\!/_{6+++}\rho_+\Psi_+^0,\\
\partial\!\!\!/_6(\rho_+\Psi_+^2 e_2) &= \partial\!\!\!/_{6-+-}(\rho_+\Psi_+^2 e_2)\\
\partial\!\!\!/_6(\rho_+\Psi_+^4 e_4) &= \partial\!\!\!/_{6--+}(\rho_+\Psi_+^4 e_4) \qquad (18)
\end{aligned}$$

(no parentheses are needed in the first of these equations (lepton term), for nonassociativity only becomes an issue on the quark terms). For any real variables x and y, and differentiable f: $(\partial_x + i\partial_y)f(x+iy) = 0$. Therefore, ignoring $\mathbf{R}^{1,3}$ coordinates, if

$$\begin{aligned}\Psi_+^0 &= \Psi_+^0(x_4+ix_8, x_5+ix_6, x_7+ix_9),\\ \Psi_+^1 &= \Psi_+^1(x_4+ix_8, x_5-ix_6, x_7-ix_9),\\ \Psi_+^2 &= \Psi_+^2(x_4-ix_8, x_5+ix_6, x_7-ix_9),\\ \Psi_+^4 &= \Psi_+^4(x_4-ix_8, x_5-ix_6, x_7+ix_9),\end{aligned} \tag{19}$$

then

$$\partial\!\!\!/_{0,6}(\rho_+\Psi) = 0. \tag{20}$$

The antimatter fields of $\rho_-\Psi$ would have functional dependencies conjugate to those above. Any fluctuation from these would give rise to unobserved matter-antimatter mixing.

Under $U(3)$ the lepton term Ψ_+^0 is supposed invariant, but its 3 complex coordinates in (19) are not. In making $U(3)$ a local gauge symmetry, dependent upon $\mathbf{R}^{1,3}$ coordinates, the complex coordinates of Ψ_+^0 also acquire a functional dependence on $\mathbf{R}^{1,3}$. The orbit of $U(3)$ is S^5, the 5-sphere. Because Ψ_+^0 is dependent on 3 complex coordinates, and not 6 real, this precludes a variation of Ψ_+^0 by even so much as a phase factor under $U(3)$. It would seem then that the colorless lepton term Ψ_+^0 must be independent entirely of the color-carrying coordinates of $\mathbf{R}^{0,6}$.

The complex triple associated with Ψ_+^1 in (19) has a more complicated SU(3) transformation, further complicated by the fact that Ψ_+^1 is itself simultaneously transformed. However, Ψ_+^1 is invariant under the action of the SU(2) subgroup of SU(3) that leaves e_1 and e_5 invariant. Following the same reasoning used above we now conclude that Ψ_+^1 must be independent, not of all of $\mathbf{R}^{0,6}$ as was Ψ_+^0, but of x_r, r=5,6,7,9.

In general we may now conclude, inorder to preserve (20), that

$$\begin{aligned}\Psi_+^0 &= \Psi_+^0(x_\mu, ..., ..., ...),\\ \Psi_+^1 &= \Psi_+^1(x_\mu, x_4+ix_8, ..., ...),\\ \Psi_+^2 &= \Psi_+^2(x_\mu, ..., x_5+ix_6, ...),\\ \Psi_+^4 &= \Psi_+^4(x_\mu, ..., ..., x_7+ix_9),\end{aligned} \tag{21}$$

where (,...) indicates independence of the complex coordinate in that slot, and x_μ denote the coordinates of $\mathbf{R}^{1,3}$.

Does any of this have anything to do with string theory? I confess myself not a string theorist, so I can not supply a definitive answer to that question. String theory uses $\mathbf{R}^{1,9}$, and it deals with the extra 6 dimensions by balling them up into a complex 3-manifold too small to be observed. My route to $\mathbf{R}^{1,9}$ is certainly different, but in requiring (20) the space $\mathbf{R}^{0,6}$ is forced to appear in the guise of a complex 3-space. It has not yet been investigated if some specific compactification is required of the model, much less if there is an associated $SU(3)$ holonomy group [13]. As to its unobservability, everything in this model (specifically quarks and $\mathbf{R}^{0,6}$) associated with the octonion units $e_p, p = 1, ..., 6$ (also associated with nonassociativity) is unobserved. There may be some nice algebraic/quantum mechanical explanation for this, but even so one finds such subtlety is generally manifested by more prosaic explanations as well, like infrared slavery, and, presumably, compactification.

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