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Integrability in N=2 Gauge Theory: A Proof

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Abstract

The holomorphic prepotential of ultraviolet finite N=2 supersymmetric gauge theories is obtained by a partial twisting of N=1 gauge theory in six dimensions, compactified on $\mathbb{R}^4 \times \mathbf{T}^2$. We show that Ward identities for the conserved chiral $\mathcal{R}$-symmetry in these theories generate a set of constraints on the correlation functions of chiral ring operators. These correlators depend only on the coordinates of the $\mathbf{T}^2$, and the constraints are analogs of the Knihnik-Zamolodchikov-Bernard equations at the critical level.

1 Introduction

There is by now a great deal of evidence [1, 2, 3, 4, 5, 6] that integrability underlies the structure [7] of N=2 supersymmetric gauge theory. In a recent article [5], the first author made a number of remarks and conjectures about the nature of this integrability. Among these were the following:[1]

[1] A discussion of some of these matters from a rather different perspective may be found in [6].

1.) An integrable system related to the finite N=2 model with an adjoint hypermultiplet (softly broken N=4 gauge theory) was found by Donagi and Witten (explicitly for SU(N)), in the context of Hitchin's integrable system [8]. A more explicit but equivalent description of the integrable model organizing the effective theory is the elliptic Calogero-Moser model [9]

$$H_2 = \tfrac{1}{2}p^2 + \tfrac{1}{2}\mu^2 \sum_\alpha \wp(\alpha \cdot q|\tau) \ , \tag{1}$$

where μ is the adjoint hypermultiplet mass, α are the roots of the Lie algebra $\mathbf{g}$ of the gauge group G, and $\wp$ is the Weierstrass function. The microscopic coupling constant of the gauge theory appears in the modulus of the torus $\tau = \frac{\theta}{2\pi} + \frac{4\pi i}{e^2}$. In the limit $\tau \to i\infty$, $\mu \to \infty$, together with a shift of $\vec{q}$, this integrable system degenerates to the affine Toda lattice, which was found in [1, 2, 3] to govern the pure N=2 gauge theory that arises in the infrared limit.

2.) Donagi and Witten [4] pointed out that the microscopic gauge coupling τ should be part of the data that specifies the integrable system, namely the modulus of the torus on which the spectral parameter lives. In ultraviolet finite N=2 theories, the non-renormalization of the action allows precise enough control over the theory that a proof of integrability should be possible. A number of ideas about how this might transpire were presented in [5]. In particular, it was proposed that the requirement of finiteness should arise via anomaly cancellation for the effective theory on the spectral parameter torus, and that the periodicity of the potential (1) could arise from considerations of a higher-dimensional theory.

3.) The partition function of finite N=2 gauge theories should satisfy the Knihnik-Zamolodchikov-Bernard equation on the spectral parameter torus of the integrable system, at the critical level $k = -h^\vee_{\mathbf{g}}$. The evidence for this is somewhat indirect. The Toda system that arises in the infrared limit of the softly broken N=4 model is the *twisted* affine Toda lattice; its dynamics comes from geodesic motion on the loop group $(LG)^\vee$, projected down to a homogenous space by gauging away a certain subgroup [9, 10, 11]. The appearance of the dual group in the infrared limit of the finite theory accords with the idea [12] that the effective theory should be a gauge theory of monopoles with gauge group $G^\vee$. In [1, 2, 3], the partition function of the N=2 theory was related to the WKB or Whitham-averaged Toda lattice, which must come from the semiclassical $k \to \infty$ limit of the loop group dynamics upstairs. The appearance of the dual group in the infrared is fortuitous, but in the ultraviolet one should see the microscopic gauge theory with gauge group G; hence the Calogero-Moser model (1) should involve this group. Remarkably, this interconversion of UV group G and IR group $G^\vee$ appears in work on the quantization of the Hitchin integrable system [13, 14] (which, in turn, is intimately connected to the model (1) [15, 16]). Namely, the semiclassical limit $k^\vee \to \infty$ of the W-algebra $W_{k^\vee}(\mathbf{g}^\vee)$ is the Gelfand-Dikii algebra $GD(\mathbf{g}^\vee)$; this is nothing other than the algebra of densities for the conserved integrals of motion of the IR integrable system. This algebra is dual (in a well-defined sense) to the critical level limit $k \to -h^\vee_{\mathbf{g}}$ of the W-algebra $W_k(\mathbf{g})$, which is the algebra of commuting Hamiltonians of the *quantized* Hitchin system – whose quadratic Hamiltonian on the torus is the quantized version of (1). Thus, Montonen-Olive duality would have its proper role in supersymmetric gauge theory if the ultraviolet theory were related to the *quantized* Calogero-Moser system (or equivalently the quantized Hitchin system). Interestingly, it would then also be intimately connected to Langlands duality [13], which is the context in which the foregoing duality of W-algebras first arose.

Our purpose here is to provide a proof of integrability in finite N=2 gauge theories. In the process, we will establish much of the structure put forward in [5]. We will also give considerable substance to the tantalizing parallels, described briefly in [2], between the chiral ring of N=2 supersymmetric models in two dimensions and the structure of the correlators of chiral fields in N=2 supersymmetric QCD in four dimensions.

The basic idea is remarkably simple. Starting from N=1 gauge theory in six dimensions, we reduce to four dimensions on a two-torus $\mathbf{T}^2$ of modulus τ whose volume is sent to zero. Then, following [17], we topologically twist the dynamics on the $\mathbf{T}^2$ to yield an effective dynamics in four-dimensional spacetime. The four-dimensional theory has N=2 supersymmetry. The BRST operators of the topological dynamics on $\mathbf{T}^2$ are the analytic half of the N=2 supercharges in four dimensions (the currents corresponding to the conjugate supercharges become the BRST partner of the stress tensor on the $\mathbf{T}^2$, rendering the dynamics there trivial). Thus the twisted theory computes the *holomorphic* prepotential of the gauge theory – any dependence on antiholomorphic fields is BRST-trivial (up to possible holomorphic anomalies [18]). A holomorphic dependence on the coordinate z of the $\mathbf{T}^2$ remains.

A crucial feature of this construction is the fact that the four-dimensional $\mathcal{R}$-symmetry comes from the local Lorentz symmetry on the torus $\mathbf{T}^2$. The anomaly of the $\mathcal{R}$-symmetry may thus be thought of as descending from the mixed Lorentz and gauge anomaly in 6 dimensions. Since the twisted theory is holomorphic on the torus, the chiral Ward identity of the $\mathcal{R}$-symmetry is, from the six-dimensional point of view, generated by the action of the holomorphic energy-momentum tensor of the (conformal) field theory on the torus $\mathbf{T}^2$. For suitable operators inserted on the $\mathbf{T}^2$, this action is in turn captured by the Knizhnik-Zamolodchikov-Bernard (KZB) equations on the torus. Thus, by using the chiral Ward identity of the four-dimensional theory – lifted to the six-dimensional setting – we discover the spectral curve, and link the chiral correlators of the four-dimensional theory to solutions of the (integrable) KZB equations. While we have not fully evaluated the fermion correlation functions which appear in these equations, we feel that the method deserves a separate brief outline. We will complete the determination of these correlators, and apply the resulting identities to various N=2 theories, in a future work [19].

2 Reduction from six dimensions

The field content of N=1 gauge theory in six dimensions [20, 21][2] consists of the vector multiplet (A_M, λ_A) of gauge fields and gauginos; together with a collection of matter hypermultiplets $(\phi^i, \psi^i_{\bar{A}})$ and their conjugate fields, transforming in some representations R_i of the gauge group G. We denote six-dimensional vector and spinor indices by M, A. Under reduction on $\mathbb{R}^4 \times \mathbf{T}^2$, we denote the corresponding quantities μ, α, $\dot{\alpha}$ for $\mathbb{R}^4$; and m, $a = \pm$ for $\mathbf{T}^2$. Irreducible spinors in six dimensions are Weyl spinors; compatibility with a chiral supersymmetry requires that λ_A be chiral, while $\psi_{\bar{A}}$ is antichiral. We take the metric on the $\mathbf{T}^2$ to be

$$ds^2 = \frac{L^2}{\tau_2}|dz|^2 \ , \tag{2}$$

where $dz = dx^4 + \tau dx^5$. In string theory $\tau = \tau_1 + i\tau_2$ is the expectation value of a gravitational vector multiplet U. In addition we will introduce a

[2]We adopt the gamma matrix conventions of Brink et.al. [20].

background antisymmetric tensor field B_{MN} with $\langle B\rangle = \frac{\theta}{2\pi}\epsilon_{56}$, which may be considered as the real part of the vev of another string-motivated vector multiplet T. Then a $B\wedge F\wedge F$ term in the action will induce the usual theta term upon reduction to four dimensions. For convenience we will take $T = U = \tau$ although this is probably not essential. One may then rather cavalierly disregard, say, the dependence upon the theta angle during the course of a derivation, knowing that it will in the end be restored by holomorphicity.

The reduction on $\mathbb{R}^4\times \mathbf{T}^2$ splits the vector multiplet according to

$$(A_M, \lambda_A) \to (A_\mu, A_{++}, A_{--}; \lambda_{\alpha +}, \lambda_{\dot\alpha -}) \ . \tag{3}$$

Similarly, the hypermultiplet splits as

$$(\phi, \psi_{\bar A}) \to (\phi; \psi_{\dot\alpha +}, \psi_{\alpha -}) \ . \tag{4}$$

The four-dimensional theory has an $\mathcal{R}$-charge which is nothing but the Lorentz spin on $\mathbf{T}^2$ (later we will consider the two-dimensional gauge fields A_m to have coordinate indices which do not transform under $\mathcal{R}$, and hence are unaffected by topological twisting).

We now write the bosonic part of the field theory action:

$$\begin{aligned} S \;=\; & \frac{4\pi}{e^2}\int d^4x\, d^2z\, \left[\frac{L^2}{\tau_2}(F_{\mu\nu}F^{\mu\nu}) + \frac{\tau_2}{L^2}(F_{mn}F^{mn}) + (D_mA_\mu - D_\mu A_m)^2\right] + \\ & \frac{i\theta}{2\pi}\int d^4x\, d^2z\, \epsilon^{\mu\nu\rho\sigma}F_{\mu\nu}F_{\rho\sigma} + \\ & \frac{4\pi}{e^2}\int d^4x\, d^2z\, \left[\frac{L^2}{\tau_2}(D_\mu\tilde\phi^i D^\mu\phi^i) + \frac{\tau_2}{L^2}(D_m\tilde\phi^i D^m\phi^i) + \frac{L^2}{\tau_2}M^j{}_i\tilde\phi^i\phi_j\right] . \end{aligned} \tag{5}$$

Now let us take the limit $L\to 0$, $e\to 0$, e/L fixed. We implicitly rescale A_m by a factor of L to keep it in the effective theory. This limit forces fields to be essentially flat as far as their dependence on the coordinates z, $\bar z$ of the $\mathbf{T}^2$. The complex coupling constant of the effective theory is τ. We may consider expanding around a configuration with a Wilson line expectation value

$$\langle A_{--}\rangle = v\cdot\mathbf{h} \quad , \qquad \langle A_{++}\rangle = \bar v\cdot\mathbf{h} \ . \tag{6}$$

In four-dimensional language, we have a non-zero Higgs vev; on the $\mathbf{T}^2$, we have a non-trivial flat gauge bundle. For instance, holomorphic objects will satisfy

$$\mathcal{O}(z+m+n\tau) = e^{-2\pi i(m+n\tau)v\cdot\mathbf{h}}\mathcal{O}(z)e^{2\pi i(m+n\tau)v\cdot\mathbf{h}} \ . \tag{7}$$

As explained for instance in [22], all quantities involving the Wilson line/Higgs vev v are invariant under the elliptic affine Weyl transformations $\delta v \in \Lambda^\vee + \tau\Lambda^\vee$, where $\Lambda^\vee$ is the coroot lattice.

Now consider twisting the theory. Our discussion will remain temporarily at the level of the classical action; consideration of the effects of quantum fluctuations are briefly deferred. A twist by the Lorentz spin along the $\mathbf{T}^2$ has the effect

$$\begin{array}{ccc} \lambda_{\alpha+} \to \lambda_\alpha & , & \lambda_{\dot\alpha-} \to \lambda_{\dot\alpha--} \\ \psi_{\alpha-} \to \psi_{\alpha--} & , & \psi_{\dot\alpha+} \to \psi_{\dot\alpha} \; ; \end{array} \tag{8}$$

similarly, the supersymmetry charges are shifted as

$$\begin{array}{ccc} Q_{\alpha+} \to Q_\alpha & , & Q_{\dot\alpha-} \to Q_{\dot\alpha--} \\ \bar Q_{\alpha+} \to \bar Q_{\alpha++} & , & \bar Q_{\dot\alpha-} \to \bar Q_{\dot\alpha} \; . \end{array} \tag{9}$$

As is by now standard, we wish to reinterpret the two-dimensional scalar charge Q_α as a BRST operator[3]; the current associated to the conjugate charge $\bar Q_{\alpha++}$ will be the BRST partner of the two-dimensional stress tensor. Something remarkable has happened, though; the Weyl condition in six dimensions, together with the topological twist using the two-dimensional spin, results in a set of BRST charges which are analytic in four dimensions. Having chosen the BRST operator to be Q_α, the effective action of the twisted theory has as BRST invariant content only the holomorphic part of the prepotential! For instance, the supersymmetry transformation laws

$$\begin{array}{rcl} \delta A_{++} & = & \bar\zeta^\alpha_{++}\lambda_\alpha + \bar\lambda^\alpha_{++}\zeta_\alpha \\ \delta A_{--} & = & \bar\zeta_{\dot\alpha}\lambda^{\dot\alpha}_{--} + \bar\lambda_{\dot\alpha}\zeta^{\dot\alpha}_{--} \end{array} \tag{10}$$

show that A_{++} is BRST trivial, and so the effective action of the twisted theory only depends upon the holomorphic part v of the Higgs vev, and not on $\bar v$.

Apropos a proposal of [5], it is amusing to see that one can find a graded algebra of charges in the twisted theory. Namely, the contour integrals $\oint A_{--}$, $\oint \psi^i_{\alpha--}$ are BRST invariant. We do not at present understand their utility.

[3] Alternatively, one could choose $\bar Q_{\dot\alpha}$ as the BRST operator; however, one cannot use both due to $\{\bar Q_{\dot\alpha}, Q_\alpha\} = 2P_{\dot\alpha\alpha}$.

However the even charges are associated to the vector multiplets and the odd ones to the hypermultiplets.

At this point we should discuss the effect of quantum fluctuations on our theory. We do not imagine starting from the untwisted theory in six dimensions, quantizing it, then twisting. This would not make any sense, as the original six-dimensional theory is sick in the ultraviolet. Rather, we wish to start with the twisted theory on $\mathbb{R}^4 \times \mathbf{T}^2$, and ask if it defines a reasonable theory of four-dimensional fields, carrying an additional holomorphic dependence on a parameter z which we may call the spectral parameter. Fixing the gauge bundle over the $\mathbf{T}^2$, the only relevant field fluctuations are in $\mathbb{R}^4$. However, to derive the low-energy theory we made a naive scaling analysis of the classical action. This procedure is patently wrong in the quantum theory, unless the gauge coupling e^2 does not undergo anomalous scaling – that is, we are dealing with an ultraviolet finite N=2 model.

A separate argument leads to the same conclusion. Consistently decoupling the non-holomorphic dependence on the $\mathbf{T}^2$ requires that the BRST charge square to zero. This implies a condition on the product of two supersymmetry currents in the full six-dimensional theory. The conservation of supercurrents is related by supersymmetry to the conservation of the stress tensor on the $\mathbf{T}^2$. Both will be spoiled by a mixed anomaly[4] of the form $tr\{R^2\}tr\{F^2\}$ in the six-dimensional theory, where the $tr\{R^2\}$ comes from the $\mathbf{T}^2$ and the $tr\{F^2\}$ from the $\mathbb{R}^4$. However $tr\{F^2\}$ will vanish in a theory whose matter content corresponds to a finite N=2 theory in four dimensions.

3 Ward identities

In supersymmetric QCD in four dimensions, the correlators of the lowest components of chiral superfields satisfy supersymmetric Ward identities which imply that these correlators are constant (*i.e.* independent of the location of the operators). This also remains true when instanton corrections are taken into account (see, for example [23]). These operators and their correlators may thus be thought of as defining the chiral ring of the theory. In terms of the six-dimensional theory, such *topological* correlators will produce holomorphic conformal blocks in the two-dimensional field theory on the torus.

[4]We thank J. Harvey for a discussion on these matters.

As was pointed out above, these will satisfy the KZB equation on the once-punctured torus (c.f. [24])

$$4\pi i(k+h^\vee_{\mathbf{g}})\frac{\partial}{\partial\tau}\tilde{\omega}(z,\tau,\vec{v}) = \Big[\frac{\partial^2}{\partial\vec{v}^{\,2}} - \sum_\alpha \wp(\alpha\cdot v)\mathbf{e}_\alpha\mathbf{e}_{-\alpha} - C_2(V)\eta_1(\tau)\Big]\tilde{\omega}\ . \quad (11)$$

In particular, as was conjectured in [5], the partition function of the softly broken N=4 theory should satisfy this equation in the critical level limit $k \to -h^\vee_{\mathbf{g}}$. In this section we will start with the standard form of the chiral symmetry Ward identity, and show how it can be lifted into six dimensions as the KZB equation. Consider the axial rotation $\delta\psi(y) = -i\delta\epsilon(y)\gamma^5\psi(y)$ of a single Dirac fermion ψ of mass M in representation R of a background gauge field A_μ (c.f. [25]):

$$\begin{aligned} 0 \;=\;& \partial_\mu\langle j^{\mu 5}(y)\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle + \\ & 2M\langle\bar{\psi}\gamma^5\psi(y)\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle + \\ & \sum_i\langle\mathcal{O}_1(x_1)\ldots\frac{\partial\mathcal{O}_i}{\partial\epsilon}\ldots\mathcal{O}_n(x_n)\rangle\delta(y-x_i) + \\ & \frac{iT_2(R)}{8\pi^2}\langle F\tilde{F}(y)\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle\ . \end{aligned} \quad (12)$$

Integrating over y, and replacing the insertion of the instanton number term by the corresponding derivative with respect to the theta parameter, we find

$$\begin{aligned} 0 \;=\;& 2M\langle\int d^4y\bar{\psi}\gamma^5\psi(y)\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle + \\ & \frac{\partial}{\partial\epsilon}\langle\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle + \\ & 4T_2(R)\frac{\partial}{\partial\theta}\langle\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle\ . \end{aligned} \quad (13)$$

To apply this result to the twisted six-dimensional model above, consider the relevant set of fermions (3),(4), making opposite chiral rotations for vector and hypermultiplet fields due to their opposite $\mathcal{R}$-charge. Replace γ^5 in the first term by $\sigma^3\Gamma^7$. The analog of the mass term M comes from two sources: The Higgs vev v for all the fields, and the flavor mass $M^i{}_j$ for the hypermultiplets (which, if one wants, may be thought of as the vev of a non-dynamical background flavor gauge field). Then (13) becomes

$$4\pi[2C_2(G) - \sum_i T_2(R_i)]\ \frac{\partial}{\partial\tau}\langle\mathcal{O}_1(x_1)\ldots\mathcal{O}_n(x_n)\rangle =$$

$$
\begin{aligned}
\langle \int d^4y \Big(\bar\lambda^\alpha_{++} A_{--} \lambda_\alpha + \bar\psi^i_{\dot\alpha ++} A_{--} \psi^{\dot\alpha}_i \Big) \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle + \\
\langle \int d^4y (M_{--})^j{}_i \bar\psi^i_{\dot\alpha ++} \psi^{\dot\alpha}_j \quad \mathcal{O}_1(x_1) \ldots \mathcal{O}_n(x_n) \rangle + \\
\sum_i \langle \mathcal{O}_1(x_1) \ldots \frac{\partial \mathcal{O}_i}{\partial \epsilon} \ldots \mathcal{O}_n(x_n) \rangle \ .
\end{aligned}
\tag{14}
$$

We have anticipated that the result must be BRST invariant by keeping only the holomorphic contributions on $\mathbf{T}^2$.

Now let us interpret the various terms in (14), and compare with equation (11). On the LHS we have the derivative with respect to the modulus of the torus; encouragingly, its coefficient in (14) vanishes for a finite theory – the corresponding KZB-like equation is at the 'critical level'. Of course, physically this just means that the $\mathcal{R}$-symmetry is not violated by instantons, and remains unbroken in the full quantum theory. Now specialize the operators $\mathcal{O}_i$ to the chiral ring of the four-dimensional N=2 theory. These are fields whose correlators have no x-dependence; however in general they will have holomorphic z-dependence. The last term in (14) then measures the two-dimensional Lorentz spins of all the fields (recall that the $\mathcal{R}$-symmetry transformation is just the Lorentz rotation on the $\mathbf{T}^2$), which for chiral ring operators is the same as the holomorphic conformal dimension. Thus two respective terms in (11) and (14) match, since $C_2(V)$ in (11) is just the holomorphic conformal dimension of the operator at the puncture.

What about the other two terms? After specializing to the chiral ring correlators, all the terms in (14) are independent of $\mathbb{R}^4$, so we may interpret them as equations on the two-dimensional spectral torus. The first term on the RHS may, using the equation of motion, be replaced by $A_{--}\partial^2_{++}A_{--}$ (always up to BRST artifacts); contour integrating over the location z of this operator on the $\mathbf{T}^2$ gives $\oint (\partial_{++}A_{--})^2$, which is an insertion of the two-dimensional conjugate momentum to A_{--} squared. A piece of this is $\partial^2/\partial v^2$. Hence to identify the KZB equation, we have only to compute the result of the flavor mass insertion – the second term on the RHS of (14). We confine ourselves here to a few general comments. The correlation function

$$
\langle \oint dz \int d^4y (M_{--})^j{}_i \bar\psi^i_{\dot\alpha ++} \psi^{\dot\alpha}_j(y,z) \ \mathcal{O}_1(z_1) \ldots \mathcal{O}_n(z_n) \rangle \tag{15}
$$

must be covariant under the shifts $z \to z + m + n\tau$ and $\vec{v} \to \vec{v} + \vec{r}_1 + \tau \vec{r}_2$ for $\vec{r}_1, \vec{r}_2 \in \Lambda^\vee$. It must have singularities when the z_i collide. These properties are shared by the KZB equation (11) and its multipuncture counterparts. We expect them to arise from a careful evaluation of the fermion propagator on the torus in the presence of a background gauge field. Of course, on general grounds, the Ward identity (14) *must* be the Virasoro Ward identity on $\mathbf{T}^2$, since this is the effect of global chiral rotations on $\mathbb{R}^4$ which are local Lorentz rotations on $\mathbf{T}^2$.

Thus we have found a partially topologically twisted N=1 gauge theory in six dimensions which computes the holomorphic prepotential of N=2 gauge theory in four dimensions. The dependence on the extra $\mathbf{T}^2$ of the reduction is holomorphic, the extra variable becoming a spectral parameter. We also found a set of Ward identities satisfied by the correlation functions of chiral ring operators in the theory which bear a remarkable similarity to the Knizhnik-Zamolodchikov-Bernard equations on $\mathbf{T}^2$. If these are not the KZB equations, they are what replace them in the context of N=2 gauge theory.

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