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T-DUALITY AND NON-LOCAL REALIZATIONS OF SUPERSYMMETRY IN STRING THEORY [1]

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ABSTRACT

We study non-local realizations of extended worldsheet supersymmetries and the associated space-time supersymmetries which arise under a T-duality transformation. These non-local effects appear when the supersymmetries do not commute with the isometry with respect to which T-duality is performed.

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1 Introduction

In string theory, a T-duality transformation is usually performed with respect to a space-time coordinate (say θ), provided the massless background fields are invariant under translations in the θ-direction. The dual theories are different space-time manifestations of the same superconformal field theory. To study the full theory, one has to go beyond the massless background fields and find how other objects and vertex operators in the theory map under duality. It so happens that when an object is not invariant under a θ-translation, then in the dual theory, it is always realized non-locally[1]. This effect can be easily studied when T-duality is implemented by a canonical transformation in the worldsheet theory[2]. Of particular interest is the behaviour of extended worldsheet supersymmetries and their associated target space supersymmetries under a T-duality transformation. When the supersymmetry charges are invariant under translations in the coordinate θ, then a duality with respect to θ does not give rise to non-local effects and the supersymmetries are preserved[3, 4, 5]. In [6] (see also[7]), we studied the general situation where the supercharges can depend on θ and addressed the issue of non-local realizations of supersymmetry. This talk contains a summary of the results appearing in this paper and is organised a follows: First, we formulate T-duality as a canonical transformation in an $N = 1$ supersymmetric non-linear σ-model. Then we consider theories with extended supersymmetry on the worldsheet and obtain the non-local objects which replace the θ-dependent complex structures in the dual theory. Using these results, we investigate the effects of this non-locality on the associated target space supersymmetry.

2 T-Duality as a Canonical Transformation in Supersymmetric Theories

Let us consider massless bosonic background fields G_{MN}, B_{MN} $(M, N = 1, ..., D)$ and Φ, which do not depend on one of the target space coordinates, denoted by $X^1 = \theta$, but may have a dependence on the remaining coordinates $X^{i+1} = x^i; i = 1, ..., D-1$. Under a T-duality transformation with respect to θ, the background fields G_{MN} and B_{MN} transform as

$$\begin{aligned} \widetilde{G}_{\theta\theta} = G_{\theta\theta}^{-1}, \qquad (\widetilde{G} \pm \widetilde{B})_{\theta i} = \mp G_{\theta\theta}^{-1}(G \pm B)_{\theta i}, \\ (\widetilde{G} + \widetilde{B})_{ij} = (G + B)_{ij} - G_{\theta\theta}^{-1}(G - B)_{\theta i}(G + B)_{\theta j}. \end{aligned} \tag{1}$$

To write down the transformation of the torsionful connections $\Omega_{MN}^{\pm K} = \Gamma_{MN}^{K} \pm \frac{1}{2} G^{KL} H_{LMN}$ under duality, we introduce two $D \times D$ matrices $Q_\pm$ given by [3]:

$$Q_\pm = \begin{pmatrix} \mp G_{\theta\theta} & \mp (G \mp B)_{\theta i} \\ 0 & 1_{D-1} \end{pmatrix}. \tag{2}$$

Using these, the transformation under duality of the metric and the torsionful connections can be written as

$$\widetilde{G}^{-1} = Q_- G^{-1} Q_-^T = Q_+ G^{-1} Q_+^T, \tag{3}$$

$$\widetilde{\Omega}^{\pm M}_{NK} = (Q_\mp^{-1})^{N'}_N \, (Q_\pm^{-1})^{K'}_K \, (Q_\mp)^M_{M'} \, \Omega^{\pm M'}_{N'K'} - \delta^i_K \, (\partial_i \, Q_\mp \, Q_\mp^{-1})^M_N. \tag{4}$$

Now, consider the bosonic non-linear σ-model and let p_θ denote the momentum canonically conjugate to θ. The duality transformations (1) then follow from the canonical transformation

$$\widetilde{\theta}' = -p_\theta \, , \qquad \widetilde{p}_\theta = -\theta' \, , \qquad \widetilde{x}^i = x^i \, . \tag{5}$$

It is clear from the above that the relation between θ and $\widetilde{\theta}$ is, in general, non-local.

In the following, we want to generalize the above procedure to the case of $N = 1$ supersymmetric non-linear σ-models defined by the action:

$$\begin{aligned} S &= \frac{1}{2} \int d^2\sigma [(G_{MN} + B_{MN}) \partial_+ X^M \partial_- X^N - i \psi_+^M G_{MN} (\delta^N_K \partial_- + \Omega^{+N}_{LK} \partial_- X^L) \psi_+^K \\ &- i \psi_-^M G_{MN} (\delta^N_K \partial_+ + \Omega^{-N}_{LK} \partial_+ X^L) \psi_-^K + \frac{1}{2} \psi_+^M \psi_+^N \psi_-^K \psi_-^L R_{MNKL}(\Omega^-)]. \end{aligned} \tag{6}$$

Here, $R_{MNKL}(\Omega^\pm)$ are the curvature tensors corresponding to the torsionful connections $\Omega^{\pm M}_{NK}$. The above action has a default $(1,1)$ supersymmetry under which the fields transform as

$$\delta_\mp X^M = \pm i \epsilon_\mp \psi_\pm^M, \quad \delta_\mp \psi_\pm^M = \pm \partial_\pm X^M \epsilon_\mp, \quad \delta_\mp \psi_\mp^M = \mp i \psi_\mp^N \epsilon_\mp \Omega^{\pm M}_{NK} \psi_\pm^K. \tag{7}$$

To preserve this $N = 1$ supersymmetry under duality, we have to supplement the canonical transformation (5) (where p_θ is now defined using the $N = 1$ supersymmetric action) with the appropriate transformations of the worldsheet fermions. In terms of the bosonic and fermionic coordinates, the resulting canonical transformation can now be written as

$$\widetilde{\psi}_\pm^M = Q^M_{\pm N} \psi_\pm^N, \qquad \partial_\pm \widetilde{\theta} = Q^\theta_{\pm M} \, \partial_\pm X^M + i \psi_\pm^j \, \partial_j Q^\theta_{\pm M} \, \psi_\pm^M. \tag{8}$$

Using the $N = 1$ superfields Φ^M, this takes the form

$$D_\pm \, \widetilde{\Phi}^M = Q^M_{\pm N}(\Phi) \, D_\pm \, \Phi^N. \tag{9}$$

The conservation of the θ-isometry current leads to $\partial_+ \partial_- \widetilde{\theta} = \partial_- \partial_+ \widetilde{\theta}$. This implies that, in spite of the non-local relation between θ and $\widetilde{\theta}$, on shell, the dual coordinate is a local function of the worldsheet coordinates $\sigma^\pm$.

Since the fermion couplings in (6) are entirely determined by the $N = 1$ supersymmetry, the canonically transformed action has the same form as the original action (6) with the backgrounds G_{MN}, B_{MN} replaced by their dual counterparts as given by (1). Now, comparing the two actions, we obtain a compact expression for the transformation of the generalized curvature tensor:

$$Q^{M'}_{+M} Q^{N'}_{+N} Q^{K'}_{-K} Q^{L'}_{-L} \widetilde{R}_{MNKL}(\widetilde{\Omega}^-) = R_{MNKL}(\Omega^-) - 2 G^{-1}_{\theta\theta} \partial_{[N} Q^\theta_{+M]} \partial_{[L} Q^\theta_{-K]}. \tag{10}$$

3 T-Duality and Non-Local Extended Supersymmetry on the Worldsheet

Let J^M_N denote a complex structure on the target space ($J^2 = -1$) and define $\psi_\pm^{(J)M} = J^M_{\pm N}\psi^N_\pm$. The invariance of the action (6), under the replacement $\psi^M_\pm \to \psi_\pm^{(J)M}$, requires that $J^T G J = G$ and $\nabla^\pm J = 0$. Here, the covariant derivatives contain the torsionful connections. If these relations hold, then the theory admits a second set of supersymmetry transformations, which are obtained from the $N = 1$ transformations (7) by the same replacement $\psi^M_\pm \to \psi_\pm^{(J)M}$.

The effect of T-duality on the extended worldsheet supersymmetries can be studied by requiring that this second set of supersymmetry transformations of the original theory imply a similar set of transformations for the dual theory. When $\partial_\theta J$ is not necessarily zero, we find that under duality the complex structures $J_\pm(\theta, x^i)$ transform to non-local objects $\widetilde{J}_\pm$ given by

$$\widetilde{J}_\pm([\widetilde{\theta}, x^i], x^i) = Q_\pm J_\pm(\theta[\widetilde{\theta}, x^i], x^i) Q_\pm^{-1}. \tag{11}$$

Here, $\theta[\widetilde{\theta}, x^i]$ is the usual notation for the functional dependence of θ on $\widetilde{\theta}$ and x^i with the explicit relation given by the second equation in (8). Note that when $\partial_\theta J = 0$, this reduces to the known transformation of J as obtained in [3, 4]. However, in general, $\widetilde{J}_\pm$ has a non-local dependence on the coordinates of the dual target space $\{\widetilde{X}^M\} = \{\widetilde{\theta},\, x^i\}$. The condition of the covariant constancy of the complex structure now gets modified to

$$\begin{aligned} \partial_\theta \widetilde{J}^M_{\pm N} + \widetilde{G}^{-1}_{\widetilde{\theta}\widetilde{\theta}} \left(\widetilde{\Omega}^{\pm M}_{\widetilde{\theta} L}\, \widetilde{J}^L_{\pm N} - \widetilde{J}^M_{\pm L}\, \widetilde{\Omega}^{\pm L}_{\widetilde{\theta} N} \right) = 0, \\ \widetilde{\nabla}^\pm_i\, \widetilde{J}^M_{\pm N} \pm (\widetilde{G} \pm \widetilde{B})_{\widetilde{\theta} i}\, \partial_\theta \widetilde{J}^M_{\pm N} = 0. \end{aligned} \tag{12}$$

Note that these equations contain derivatives of $\widetilde{J}_\pm$ with respect to θ and not with respect to the natural coordinate on the dual space, which is $\widetilde{\theta}$.

Even when the extended supersymmetry becomes non-local under duality, the extended superconformal algebra remains unchanged. However, this algebra is now realized in terms of non-local supercharges, and the representation becomes non-local. Such non-local representations in a class of conformal field theories were constructed in terms of parafermions in [9]. There are several explicit examples known in which a part of the extended supersymmetry becomes non-local under duality [8, 7, 5].

4 Implications for Target Space Supersymmetry

A configuration of the bosonic background fields admits $N = 1$ space-time supersymmetry provided the supersymmetric variations of the gravitino (Ψ_M) and dilatino (λ) fields vanish.

Let us consider $\delta\Psi_M = 0$, which leads to the Killing spinor equation

$$\delta\Psi_M = \partial_M \eta + \frac{1}{4}\left(\omega_M^{AB} - \frac{1}{2} H_M{}^{AB}\right)\gamma_{AB}\,\eta = 0. \tag{13}$$

Here, ω_M^{AB} is the spin connection and A, B are tangent space indices. When the target space supersymmetry is a consequence of an extended supersymmetry on the worldsheet, then the complex structure J and the Killing spinor η are related by [10]

$$J_{+N}^{M} = \bar{\eta}\,\gamma^{M}{}_{N}\,\eta. \tag{14}$$

The Killing spinor condition then implies that $\nabla_M^+ J_{+N}^K = 0$. Equation (14) can be used to study the effect of T-duality on space-time supersymmetry. In the following, we describe the three cases that may arise:

Case 1: Here, $\partial_\theta \eta = 0$, which implies $\partial_\theta J_+ = 0$. In this case, η is invariant under duality (up to a possible local Lorentz transformation). The Killing spinor condition and hence the supersymmetry are preserved.

Case 2: If $\partial_\theta J_\pm \neq 0$, then $\partial_\theta \eta \neq 0$. In this case, the extended worldsheet supersymmetry is non-locally realized after duality. Equation (14) then implies that in the dual theory η is replaced by a non-local object $\widetilde{\eta}$, given by

$$\widetilde{\eta}([\widetilde{\theta}, x], x) = \eta(\theta[\widetilde{\theta}, x], x). \tag{15}$$

The Killing spinor condition is modified to

$$\begin{aligned} \partial_\theta\,\widetilde{\eta} + \tfrac{1}{4}\widetilde{G}^{-1}_{\widetilde{\theta}\widetilde{\theta}}\,\left(\widetilde{\omega}_{\widetilde{\theta}}^{AB} - \tfrac{1}{2}\widetilde{H}_{\widetilde{\theta}}{}^{AB}\right)\gamma_{AB}\,\widetilde{\eta} = 0, \\ \partial_i\widetilde{\eta} + \tfrac{1}{4}\left(\widetilde{\omega}_i^{AB} - \tfrac{1}{2}\widetilde{H}_i{}^{AB}\right)\gamma_{AB}\,\widetilde{\eta} + (\widetilde{G} + \widetilde{B})_{\widetilde{\theta} i}\,\partial_\theta\,\widetilde{\eta} = 0. \end{aligned} \tag{16}$$

This indicates that the target space supersymmetry is no longer realized in the conventional way.

Case 3: The only other possibility is when η depends on θ in such a way that the θ-dependences on the right-hand side of (14) cancel out, giving rise to a θ-independent J. In this case, in the dual theory, the extended worldsheet supersymmetry is locally realized while the associated target space supersymmetry has a non-local realization.

The realization of supersymmetry in cases 2 and 3 is highly non-conventional. Due to their non-local nature, these transformations make sense only when the coordinates are restricted to the string worldsheet, and not at a generic space time point. Since the background fields are invariant under supersymmetry, the non-locality does not show up as long as we are looking at the vacuum configurations. However, the supersymmetry will be non-locally realized on the spectrum of fluctuations around these backgrounds, which are the relevant quantum fields for the low-energy theory.

In string theory, equivalence under T-duality is a consequence of the existence of both momentum and winding modes associated with the compact coordinate θ. It is well known that under duality these modes are interchanged: The conserved momentum P_θ and the winding number L_θ associated with the compact coordinate θ (with non-trivial π_1) are given by $P_\theta = \int_0^{2\pi} d\sigma p_\theta$ and $L_\theta = \theta(\sigma = 2\pi) - \theta(\sigma = 0)$. Then, from the canonical transformation (8), it follows that $\widetilde{P}_\theta = -L_\theta$ and $\widetilde{L}_\theta = -P_\theta$. Since the momentum and winding modes are associated with the worldsheet coordinates τ and σ, respectively, their interchange under duality is the origin of the non-local relationship between θ and $\widetilde{\theta}$. This can be easily seen when the backgrounds are flat and one can write $\theta = \theta_L + \theta_R$, whereas $\widetilde{\theta} = \theta_L - \theta_R = \int d\sigma^+ \partial_+ \theta - \int d\sigma_- \partial_- \theta$. As for the behaviour of supersymmetry, note that the parameter $\eta(\theta, x)$ is sensitive to the string momentum and winding modes associated with θ. The non-locality in the dual theory arises from the fact that the momentum and winding modes of the dual string enter $\widetilde{\eta}$ not through $\widetilde{\theta}$ (which would have resulted in a local spinor $\widetilde{\eta}(\widetilde{\theta})$), but through the original coordinate θ.

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