Non-Abelian Antibrackets

Jorge Alfaro
Fac. de Fisica
Universidad Catolica de Chile
Casilla 306, Santiago 22, Chile

Poul H. Damgaard
The Niels Bohr Institute
Blegdamsvej 17
DK-2100 Copenhagen
Denmark

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Abstract

The Δ-operator of the Batalin-Vilkovisky formalism is the Hamiltonian BRST charge of Abelian shift transformations in the ghost momentum representation. We generalize this Δ-operator, and its associated hierarchy of antibrackets, to that of an arbitrary non-Abelian and possibly open algebra of any rank. We comment on the possible application of this formalism to closed string field theory.

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In order to see how the conventional antibracket formalism of Batalin and Vilkovisky [1] can be generalized, it is important to have a fundamental principle from which this formalism can be derived. As has been discussed in a series of papers [2, 3][1], this principle is that Schwinger-Dyson BRST symmetry [5] must be imposed on the full path integral.

Schwinger-Dyson BRST symmetry can be derived from the local symmetries of the given path integral measure. When the measure is flat, the relevant symmetry is that of local shifts, and the resulting Schwinger-Dyson BRST symmetry leads directly to a quantum Master Equation on the action S which is exponentiated inside the path integral. This action depends on two new sets of ghosts and antighosts, c^A and ϕ^*_A [2]. The conventional Batalin-Vilkovisky formalism for an action S^{BV} follows if one substitutes $S[\phi,\phi^*,c] = S^{BV}[\phi,\phi^*] - \phi^*_A c^A$ and integrates out the ghosts c^A. The so-called "antifields" of the Batalin-Vilkovisky formalism are simply the Schwinger-Dyson BRST antighosts ϕ^*_A [2].

It is of interest to see what happens if one abandons[2] the assumption of flat measures for the fields ϕ^A, and if one does not restrict oneself to local transformations that leave the functional measure invariant. Some steps in this direction were recently taken in ref. [3]. One here exploits the reparametrization invariance encoded in the path integral by performing field transformations $\phi^A = g^A(\phi', a)$ depending on new fields a^i. It is natural to assume that these transformations form a group, or more precisely, a quasigroup [7]. The objects

$$u^A_i \equiv \left.\frac{\delta^r g^A}{\delta a^i}\right|_{a=0} \tag{1}$$

are gauge generators of this group. They satisfy

$$\frac{\delta^r u^A_i}{\delta\phi^B}u^B_j - (-1)^{\epsilon_i\epsilon_j}\frac{\delta^r u^A_j}{\delta\phi^B}u^B_i = -u^A_k U^k_{ij} \ , \tag{2}$$

where the U^k_{ij} are structure "coefficients" of the group. They are supernumbers with the property

$$U^k_{ij} = -(-1)^{\epsilon_i\epsilon_j}U^k_{ji} \ . \tag{3}$$

In ref. [3], specializing to compact supergroups for which $(-1)^{\epsilon_i}U^i_{ij} = 0$, the following Δ-operator was derived:

$$\Delta G \equiv (-1)^{\epsilon_i}\left[\frac{\delta^r}{\delta\phi^A}\frac{\delta^r}{\delta\phi^*_i}G\right]u^A_i + \frac{1}{2}(-1)^{\epsilon_i+1}\left[\frac{\delta^r}{\delta\phi^*_j}\frac{\delta^r}{\delta\phi^*_i}G\right]\phi^*_k U^k_{ji} \ . \tag{4}$$

When the coefficients U^k_{ij} are constant, this Δ-operator is nilpotent: $\Delta^2 = 0$. As noted by Koszul [8], and rediscovered by Witten [9], one can define an antibracket (F,G) by the rule

$$\Delta(FG) = F(\Delta G) + (-1)^{\epsilon_G}(\Delta F)G + (-1)^{\epsilon_G}(F,G) \ . \tag{5}$$

Explicitly, for the case above, this leads to the following new antibracket [3]:

$$(F,G) \equiv (-1)^{\epsilon_i(\epsilon_A+1)}\frac{\delta^r F}{\delta\phi^*_i}u^A_i\frac{\delta^l G}{\delta\phi^A} - \frac{\delta^r F}{\delta\phi^A}u^A_i\frac{\delta^l G}{\delta\phi^*_i} + \frac{\delta^r F}{\delta\phi^*_i}\phi^*_k U^k_{ij}\frac{\delta^l G}{\delta\phi^*_j} \tag{6}$$

This antibracket is statistics-changing, $\epsilon((F,G)) = \epsilon(F) + \epsilon(G) + 1$, and has the following properties:

$$\begin{aligned}(F,G) &= (-1)^{\epsilon_F\epsilon_G+\epsilon_F+\epsilon_G}(G,F) \\ (F,GH) &= (F,G)H + (-1)^{\epsilon_G(\epsilon_F+1)}G(F,H)\end{aligned} \tag{7}$$

[1] The case of extended BRST symmetry is derived in ref. [4].

[2] See the 2nd reference in [2]. This is related to the covariant formulations of the antibracket formalism [6].

$$(FG,H) \;=\; F(G,H)+(-1)^{\epsilon_G(\epsilon_H+1)}(F,H)G \tag{8}$$
$$0 \;=\; (-1)^{(\epsilon_F+1)(\epsilon_H+1)}(F,(G,H))+\text{cyclic perm.}\; . \tag{9}$$

Furthermore,

$$\Delta(F,G)=(F,\Delta G)-(-1)^{\epsilon_G}(\Delta F,G)\ . \tag{10}$$

The Δ given in eq. (4) is clearly a non-Abelian generalization of the conventional Δ-operator of the Batalin-Vilkovisky formalism.

We shall now show how to extend this construction to the general case of non-linear and open algebras. Recently, interest in more complicated algebras such as strongly homotopy Lie algebras [10] has arisen in the context of string field theory [11].

Consider the quantized Hamiltonian BRST operator Ω for first-class constraints with an arbitrary, possibly open, gauge algebra [12].[3] Apart from a set of phase space operators Q^i and P_i, introduce a ghost pair $\eta^i, \mathcal{P}_i$. They have Grassmann parities $\epsilon(\eta^i)=\epsilon(\mathcal{P}_i)=\epsilon(Q^i)+1\equiv\epsilon_i+1$, and are canonically conjugate with respect to the usual graded commutator:

$$[\eta^i,\mathcal{P}_j]=\eta^i\mathcal{P}_j-(-1)^{(\epsilon_i+1)(\epsilon_j+1)}\mathcal{P}_j\eta^i=i\delta^i_j\ . \tag{11}$$

In addition $[\eta^i,\eta^j]=[\mathcal{P}_i,\mathcal{P}_j]=0$. The quantum mechanical BRST operator can then be written in the form of a $\mathcal{P}\eta$ normal-ordered expansion in powers of the $\mathcal{P}$'s [12]:

$$\Omega=G_i\eta^i+\sum_{n=1}^{\infty}\mathcal{P}_{i_n}\cdots\mathcal{P}_{i_1}U^{i_1\cdots i_n}\ . \tag{12}$$

Here

$$U^{i_1\cdots i_n}=\frac{(-1)^{\epsilon^{i_1\cdots i_{n-1}}_{j_1\cdots j_n}}}{(n+1)!}U^{i_1\cdots i_n}_{j_1\cdots j_{n+1}}\eta^{j_{n+1}}\cdots\eta^{j_1}\ , \tag{13}$$

and the sign factor is defined by:

$$\epsilon^{i_1\cdots i_{n-1}}_{j_1\cdots j_n}=\sum_{k=1}^{n-1}\sum_{l=1}^{k}\epsilon_{i_l}+\sum_{k=1}^{n}\sum_{l=1}^{k}\epsilon_{j_l}\ . \tag{14}$$

The $U^{i_1\cdots i_1}_{j_n\cdots j_{n+1}}$'s are generalized structure coefficients. For rank-1 theories the expansion ends with the 2nd term, involving the usual Lie algebra structure coefficients U^k_{ij}. The number of terms that must be included in the expansion of eq. (12) increases with the rank. By construction $\Omega^2=0$.

The functions G_i appearing in eq. (12) are the constraints. In the quantum case they satisfy the constraint algebra

$$[G_i,G_j]\;=\;iG_kU^k_{ij}\ . \tag{15}$$

We choose these to be the ones associated with motion on the supergroup manifold defined by the transformation $\phi^A=g^A(\phi',a)$.

When considering representations of the (super) Heisenberg algebra (11), one normally chooses the operators to act to the right. Thus, in the ghost coordinate representation we could take

$$\mathcal{P}_j=i(-1)^{\epsilon_j}\frac{\delta^l}{\delta\eta^j}\ , \tag{16}$$

[3] For a comprehensive review of the classical Hamiltonian BRST formalism, see, *e.g.*, ref. [13].

and similarly in the ghost momentum representation we could take

$$\eta^j = i\frac{\delta^l}{\delta \mathcal{P}_j} \ . \tag{17}$$

On the other hand, the most convenient representation of the constraint G_j is [7]

$$\overleftarrow{G}_j = -i\frac{\overleftarrow{\delta}^r}{\delta\phi^A}u_j^A \ , \tag{18}$$

which involves a right-derivative *acting to the left*. Using eq. (2), $\overleftarrow{G}_j$ is seen to satisfy

$$[\overleftarrow{G}_i, \overleftarrow{G}_j] = i\overleftarrow{G}_k U_{ij}^k \ . \tag{19}$$

Since we wish Ω of eq. (12) to act in a definite way, we choose representations of the (super) Heisenberg algebra (11) that involve operators acting to the left as well. These are

$$\overleftarrow{\mathcal{P}}_j = i\frac{\overleftarrow{\delta}^r}{\delta\eta^j} \tag{20}$$

in the ghost coordinate representation, and

$$\overleftarrow{\eta}_j = i(-1)^{\epsilon_j}\frac{\overleftarrow{\delta}^r}{\delta\mathcal{P}_j} \tag{21}$$

in the ghost momentum representation. Inserting these operators into eq. (12) will give the corresponding BRST operator $\overleftarrow{\Omega}$ acting to the left. We now identify the ghost momentum $\mathcal{P}_j$ with the Lagrangian antighost ("antifield") ϕ^*_j.

As a special case, consider the operator $\overleftarrow{\Omega}$ in the case of an ordinary rank-1 super Lie algebra for which $(-1)^{\epsilon_i}U^i_{ij} = 0$. In the ghost momentum representation $\overleftarrow{\Omega}$ takes the form

$$\overleftarrow{\Omega} = (-1)^{\epsilon_i}\frac{\overleftarrow{\delta}^r}{\delta\phi^A}u_i^A\frac{\overleftarrow{\delta}^r}{\delta\phi^*_i} - \frac{1}{2}(-1)^{\epsilon_j}\phi^*_k U^k_{ij}\frac{\overleftarrow{\delta}^r}{\delta\phi^*_j}\frac{\overleftarrow{\delta}^r}{\delta\phi^*_i} \ . \tag{22}$$

One notices that the $\overleftarrow{\Omega}$ of the above equation coincides with our non-Abelian Δ-operator of eq. (4). In detail:

$$\Delta F \ \equiv \ F\overleftarrow{\Omega} \ . \tag{23}$$

For a rank-0 algebra – the Abelian case – we get, with the same identification,

$$\overleftarrow{\Omega} = (-1)^{\epsilon_A}\frac{\overleftarrow{\delta}^r}{\delta\phi^A}\frac{\overleftarrow{\delta}^r}{\delta\phi^*_A} \ . \tag{24}$$

The associated Δ-operator, defined through eq. (23) is seen to agree with the Δ of the conventional Batalin-Vilkovisky formalism [1].

We define the general Δ-operator through the identification (23) and the complete expansion

$$\overleftarrow{\Omega} = (-1)^i \frac{\overleftarrow{\delta^r}}{\delta\phi^A} u_i^A \frac{\overleftarrow{\delta^r}}{\delta\phi_i^*} + \sum_{n=1}^{\infty} \phi_{i_n}^* \cdots \phi_{i_1}^* \overleftarrow{U}^{i_1\cdots i_n} . \tag{25}$$

Here

$$\overleftarrow{U}^{i_1\cdots i_n} = \frac{(-1)^{\epsilon_{j_1\cdots j_n}^{i_1\cdots i_{n-1}}}}{(n+1)!} (i)^{n+1} (-1)^{\epsilon_{j_1}+\cdots+\epsilon_{j_{n+1}}} U_{j_1\cdots j_{n+1}}^{i_1\cdots i_n} \frac{\overleftarrow{\delta^r}}{\delta\phi_{j_{n+1}}^*} \cdots \frac{\overleftarrow{\delta^r}}{\delta\phi_{j_1}^*} . \tag{26}$$

By construction we then have $\Delta^2 = 0$.

It is quite remarkable that the above derivation, based on Hamiltonian BRST theory in the operator language, has a direct counterpart in the Lagrangian path integral. The two simplest cases, that of rank-0 and rank-1 algebras have been derived in detail in the Lagrangian formalism in refs. [2, 3]. It is intriguing that completely different manipulations (integrating out the corresponding ghosts c^i, and partial integrations inside the functional integral) in the Lagrangian framework leads to these quantized Hamiltonian BRST operators. The rank-0 case, that of the conventional Batalin-Vilkovisky formalism, corresponds to the gauge generators

$$\overleftarrow{G}_A = -i \frac{\overleftarrow{\delta^r}}{\delta\phi^A} . \tag{27}$$

These are generators of translations: when the functional measure is flat, the Schwinger-Dyson BRST symmetry is generated by local translations. The non-Abelian generalizations correspond to imposing different symmetries as BRST symmetries in the path integral [3].

These non-Abelian BRST operators $\overleftarrow{\Omega}$ can be Abelianized by canonical transformations involving the ghosts [14], but the significance of this in the present context is not clear. Since in general the number of "antifields" ϕ_i^* will differ from that of the fields ϕ^A, it is obvious that u_i^A in general will be non-invertible. Even when the number of antifields matches that of fields, the associated matrix u_B^A may be non-invertible ("degenerate").[4]

Having the general Δ-operator available, the next step consists in extracting the associated antibracket. By the definition (5), this antibracket measures the failure of Δ to be a derivation. When Δ is a second-order operator, the antibracket so defined will itself obey the derivation rule (8). For higher-order Δ-operators this is no longer the case. The antibracket will then in all generality only obey the much weaker relation

$$(F, GH) = (F, G)H - (-1)^{\epsilon_G} F(G, H) + (-1)^{\epsilon_G} (FG, H) . \tag{28}$$

The relation (10) also holds in all generality. When the Δ-operator is of order three or higher, the antibracket defined by (5) will not only fail to be a derivation, but will also violate the Jacobi identity (9).

For higher-order Δ-operators one can, as explained by Koszul [8], use the failure of the antibracket to be a derivation to define *higher antibrackets*. These are Grassmann-odd analogues of Nambu brackets [17, 18]. The construction is most conveniently done in an iterative procedure, starting with the Δ-operator itself [8, 19]. To this end, introduce objects Φ_Δ^n which are defined as follows:[5]

$$\Phi_\Delta^1(A) = (-1)^{\epsilon_A} \Delta A$$

[4] In the special case where u_B^A is invertible, the transformation $\phi_A^* \to \phi_B^*(u^{-1})_A^B$ makes the corresponding Δ-operator Abelian [3], but we are not interested in that case here. See also refs. [15, 16].

[5] Note that our definitions differ slightly from ref. [8, 19] due to our Δ-operators being based on right-derivatives, while those of ref. [8, 19] are based on left-derivatives.

$$\begin{aligned}
\Phi^2_\Delta(A,B) &= \Phi^1_\Delta(AB) - \Phi^1_\Delta(A)B - (-1)^{\epsilon_A} A\Phi^1_\Delta(B) \\
\Phi^3_\Delta(A,B,C) &= \Phi^2_\Delta(A,BC) - \Phi^2_\Delta(A,B)C - (-1)^{\epsilon_B(\epsilon_A+1)} B\Phi^2_\Delta(A,C) \\
&\cdot \\
&\cdot \\
&\cdot \\
\Phi^{n+1}_\Delta(A_1,\ldots,A_{n+1}) &= \Phi^n_\Delta(A_1,\ldots,A_nA_{n+1}) - \Phi^n_\Delta(A_1,\ldots,A_n)A_{n+1} \\
&\quad -(-1)^{\epsilon_{A_n}(\epsilon_{A_1}+\cdots+\epsilon_{A_{n-1}}+1)} A_n\Phi^n_\Delta(A_1,\ldots,A_{n-1},A_{n+1}) \ .
\end{aligned} \tag{29}$$

The Φ^n_Δ's define the higher antibrackets. For example, the usual antibracket is given by

$$(A,B) \equiv (-1)^{\epsilon_A}\Phi^2_\Delta(A,B) \ . \tag{30}$$

The iterative procedure clearly stops at the first bracket that acts like a derivation. For example, the "three-antibracket" defined by $\Phi^3_\Delta(A,B,C)$ directly measures the failure of Φ^2_Δ to act like a derivation. But more importantly, it also measures the failure of the usual antibracket to satisfy the graded Jacobi identity:[6]

$$\begin{aligned}
\sum_{\text{cycl.}} (-1)^{(\epsilon_A+1)(\epsilon_C+1)}(A,(B,C)) &= (-1)^{\epsilon_A(\epsilon_C+1)+\epsilon_B+\epsilon_C}\Phi^1_\Delta(\Phi^3_\Delta(A,B,C)) \\
&\quad + \sum_{\text{cycl.}} (-1)^{\epsilon_A(\epsilon_C+1)+\epsilon_B+\epsilon_C}\Phi^3_\Delta(\Phi^1_\Delta(A),B,C) \ ,
\end{aligned} \tag{31}$$

and so on for the higher brackets.

When there is an infinite number of higher antibrackets, the associated algebraic structure is analogous to a strongly homotopy Lie algebra L_∞. The L_1 algebra is then given by the nilpotent Δ-operator, the L_2 algebra is given by Δ and the usual antibracket, the L_3 algebra by these two and the additional "three-antibracket", etc. The set of higher antibrackets defined above seems natural in closed string field theory [11], the corresponding Δ-operator being given by the string field BRST operator Q.

Having constructed the Δ-operator (and its associated hierarchy of antibrackets), it is natural to consider a quantum Master Equation of the form

$$\Delta \exp\left[\frac{i}{\hbar}S(\phi,\phi^*)\right] = 0 \ . \tag{32}$$

Using the properties of the Φ^n's defined above, we can write this Master Equation as a series in the higher antibrackets,

$$\sum_{k=1}^{\infty}\left(\frac{i}{\hbar}\right)^k \frac{\Phi^k(S,S,\ldots,S)}{k!} \ = \ 0 \ , \tag{33}$$

where each of the higher antibrackets $\Phi^k(S,S,\ldots,S)$ has k entries. The series terminates at a finite order if the associated BRST operator terminates at a finite order. For example, in the Abelian case of shift symmetry the general equation (33) reduces to $i\hbar\Delta S - \frac{1}{2}(S,S) = 0$, the Master Equation of the conventional Batalin-Vilkovisky formalism.

A solution S to the general Master Equation (33) is invariant under deformations

$$\delta S = \sum_{k=1}^{\infty}\left(\frac{i}{\hbar}\right)^{k-1} \frac{\Phi^k(\epsilon,S,S,\ldots,S)}{(k-1)!} \ , \tag{34}$$

[6] We thank K. Bering for pointing out an error in the original version of this manuscript.

where again each Φ^k has k entries, and ϵ is Grassmann-odd. One can view this as the possibility of adding a BRST variation

$$\sigma\epsilon = \sum_{k=1}^{\infty} \left(\frac{i}{\hbar}\right)^{k-1} \frac{\Phi^k(\epsilon, S, S, \ldots, S)}{(k-1)!} \tag{35}$$

to the action. Here σ is the appropriately generalized "quantum BRST operator".[7] In the case of the Abelian shift symmetry, the above σ-operator becomes $\sigma\epsilon = \Delta\epsilon + (i/\hbar)(\epsilon, S)$, which precisely equals ($(i\hbar)^{-1}$ times) the quantum BRST operator of the conventional Batalin-Vilkovisky formalism.

We note that the general Master Equation (33) and the BRST symmetry (34) has the same relation to closed string field theory [11, 21] that the conventional Batalin-Vilkovisky Master Equation and BRST symmetry has to open string field theory [9]. The rôle of the action S is then played by the string field Ψ, and the Master Equation (33) is the analogue of the closed string field equations. The symmetry (34) is then the analogue of the closed string field theory gauge transformations.

The present definition of higher antibrackets suggests the existence of an analogous hierarchy of Grassmann-even brackets based on a supermanifold and a non-Abelian open algebra – a natural generalization of Possion-Lie brackets. It should also be interesting to investigate the Poisson brackets and Nambu brackets generated by the generalized antibrackets and suitable vector fields V anticommuting with the generalized Δ-operator (and in particular certain Hamiltonian vector fields within the antibrackets), as described in the case of the usual antibracket in ref. [20].

So far our construction has been carried out in the ghost momentum representation of the super Heisenberg algebra. But the definition of an antibracket from the quantized Hamiltonian BRST operator can of course be given in different representations on the extended phase space spanned by Q^i, P_j and $\eta^i, \mathcal{P}_j$. For example, in the ghost coordinate representation, the BRST operator is a first-order operator for Abelian and true Lie algebras (with just the trivial "one-bracket" defined by it), but it becomes a higher order operator suitable for defining higher antibrackets for general open algebras. It is certainly a challenge to find the rôle played by the associated antibracket structure – in particular in the ghost momentum representation – in the Hamiltonian language.

We have restricted ourselves to field transformations $\phi^A = g^A(\phi', a)$ that do not involve the ghosts c^i or antighosts ϕ^*_i. Enlarging the transformations in this way should lead to a fully covariant formulation of these non-Abelian antibrackets and Δ-operators. It is interesting to speculate, conversely, on the meaning of the corresponding "covariant" Hamiltonian BRST operators. In fact, the analyses of ref. [6] point, together with the present observations, towards some surprising analogies in the Hamiltonian and Lagrangian formulations. We hope some of these aspects can become clarified in the future.

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