Gauga invariant reformulation and BRST quantization of the nonconfining Schwinger model

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Abstract

A new generalization of the vector Schwinger model is considered where gauge symmetry is broken at the quantum mechanical level. By proper extension of the phase space this broken symmetry has been restored. Also an equivalent first class theory is reformulated in the actual phase space using Mitra and Rajaraman's prescription [8, 9]. A BRST invariant effective action is also formulated. The new dynamical fields introduced, turn into Wess-Zumino scalar.

1 Introduction

Whenever a classical symmetry is broken at the quantum mechanical level an anomaly has come into play which threatens the gauge invariance of the theory. The mechanisms for restoration of this symmetry by anomaly cancelation are of particular interest [1, 2, 3, 4], since it is the gauge invariance that regulates the unitarity and renormalizablity of a theory. Jackiw and Rajaraman[5] shows that a two dimensional quantum field theory is possible even in the gauge non-invariant formalism. An equivalent gauge invariant version is possible as was suggested by Fadeev and Satashvilli [1]. Instead of that a new kind of quantization procedure was developed by Batalin and Fradkin(BF) [2]. But the combined formalism developed by Batalin, Fradkin and Vilkovisky [2, 3] is more powerful in deriving the covariantly gauge fixed systems with first class constraints. Fujiwara, Igararishi and Kubo (FIK) [6] finaly pointed out that the fields needed for extension of the phase space in order to make a gauge invariant theory, turn into Wess-Zumino scalars with the proper choice of gauge condition.

From first principle ,viz., formalism based on Dirac's procedure of quantization of constrained system, an anomalous theory can be made gauge invariant, as was done by Wotzasek [7]. Here also one needs some auxiliary fields whiich though extend the phase space in order to transform the constraints of a particular system from second class to first class, do not change the physical containts of the theory.

Mitra and Rajaraman [8, 9] developed a new way to convert an anomalous theory into a gauge invariant one without extension of the phase space. Their motivation was like that. If one come across a theory with constraint the ideal thing to do is to eliminate these and rewrite the theory in terms of unconstrained variables. But for varity of reason first class constraints are left and the second class constraints are treated in this way. Unfortunately elimination of full set of constraints is not an easy task. Mitra and Rajaraman's suggestion [8, 9] is to ignore half of them and convert the rest to first class constraints. Then the theory can be treated by standard gauge theoretic method. Of course the Hamiltonian has to be altered.

2 Brief Review of The Model

Recently ordinary (vector) Schwinger model is studied for a one- parameter class of regularizations [10] commonly used in the study of the anomalous *chiral* Schwinger model [5, 11] and shown to be sensible and in fact solvable for a range of values of the parameter. There is a massless boson in the spectrum except at the value which corresponds to the usual treatment of the model. As in two dimension a boson can be thought of in terms of a fermion , the fermions are not confined here. A comment should be made about this regularization. The regularization is involved when one calculates the effective action by integrating the fermion out. If one integrates out the two chiral fermions occurring in the Schwinger model one by one, one can use the regularization procedures common in dealing with the chiral Schwinger model at each of the two stages. This is how the one parameter class of regularizations associated with the chiral Schwinger model enters the vector Schwinger model. As in the case of the chiral Schwinger model, the generalized regularization preserves only global gauge invariance and violates the local invariance of the action. But we have learnt from the study of the chiral Schwinger model that this does not go against any physical principle. After all, only the global invariance is a physical symmetry in the space of states.

The Schwinger model [12] is defined by the Lagrangian density

$$\mathcal{L}_F = \bar{\psi}(i\partial\!\!\!/ - eA\!\!\!/)\psi - \frac{1}{4}F^{\mu\nu}F_{\mu\nu}, \tag{1}$$

where the Lorentz indices run over the two values $0, 1$ corresponding to a two dimensional spacetime and the rest of the notation is standard. Notice that the coupling constant e has unit mass dimension in this situation. The discussion of the model is simplified by bosonizing the fermion field ψ. This leads to a Lagrangian density involving a scalar field ϕ instead of the Dirac field ψ:

$$\mathcal{L}_B = \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + \frac{1}{2}(\dot{A}_1 - A'_0)^2 - e(\phi' A_0 - \dot{\phi}A_1) - \frac{1}{2}ae^2(A_0^2 - A_1^2). \tag{2}$$

The first piece is the kinetic energy term for the scalar field and the second is the corresponding term for the gauge field – note that there is no magnetic field strength in two dimensions. The third term represents the gauge interaction. The last term involves something new, *viz.*, an undetermined parameter a, which arises because of the regularization ambiguity [5]. To be more specific, if the left handed component of ψ is integrated out first, the regularization of the determinant contains such a parameter, as stated in [5] and demonstrated in [13, 14]. The integration over the right handed component leads to a second parameter of the same type. The final Lagrangian contains the sum of the two parameters, which has been called a here. As we shall see below, there is a specific value (zero) of a which makes the Lagrangian density (2) gauge invariant. It is this value which is normally considered. However, the explicit calculation of the bosonized action does throw up the undetermined parameter. Moreover, as discussed above and demonstrated below, no law of physics is violated if a is greater than zero. We therefore retain it and investigate the consequences. It is necessary to carry out a constraint analysis of the theory (2). The momenta corresponding to A_0, A_1, ϕ respectively are

$$\pi_0 = 0, \tag{3}$$

$$\pi_1 = \dot{A}_1 - A_0', \tag{4}$$

$$\pi_\phi = \dot{\phi} + eA_1. \tag{5}$$

The first of these is a primary constraint which is familiar in gauge theory. The other two can be used to define the time derivatives of the fields in terms of the momenta. In this way the canonical Hamiltonian density may be calculated:

$$\mathcal{H} = \frac{1}{2}(\pi_\phi - eA_1)^2 + \frac{1}{2}\pi_1^2 + \frac{1}{2}\phi'^2 + \pi_1 A_0' + e\phi' A_0 + \frac{1}{2}ae^2(A_0^2 - A_1^2). \tag{6}$$

The preservation of the constraint (3) for all time requires the vanishing of the Poisson bracket between π_0 and the Hamiltonian. This yields the secondary constraint

$$\pi_1' - e\phi' - ae^2 A_0 = 0. \tag{7}$$

This is Gauss's law for this theory. Note that if a differs from zero, this constraint makes the primary constraint (3) second class and there are no further constraints in the theory. This is the generic situation.

The exceptional situation is when $a = 0$. In this case the Poisson bracket of the two constraints (3) and (7) vanishes.One can check that the preservation in time of these constraintsdoes not produce any further constraint, so that there are two first class constraints in the theory. First class constraints of course signify gauge invariance: this situation is the conventional one. We have already found that it is an exceptional situation which, together with the fact that the Dirac brackets of A_1 and π_1 are canonical, lead to equations of motion for a scalar field with mass e. Thus there is a massive particle but there is no free fermion, *i.e.*, the fermion gets confined in this exceptional situation. The massive particle is to be interpreted as a gauge boson which has acquired mass *or* as a bound state of a fermion and an antifermion. To understand this dual interpretation, note first that the above analysis suggests that the massive particle is described by the field π_1 and is identifiable with the gauge boson. However,in view of Gauss's law, the same particle may also be described by the field ϕ and is related to the fermion. As ϕ is related linearly to the fermion current, it is natural to think of it as the field for a bound state of a fermion and ananti fermion. Thus we have a duality of descriptions.

Let us now study the generic situation $a \neq 0$. As mentioned before, the constraints (3) and (7) are second class. In fact, (7) can be solved for A_0:

$$A_0 = \frac{\pi}{ae^2}(\pi_1' - e\phi'). \tag{8}$$

Using this equation, which can be imposed strongly, one may eliminate A_0 from the theory, while its conjugate gets eliminated by virtue of (3). The remaining variables are easily seen to have canonical Dirac brackets. The Hamiltonian density (6) can be written as

$$\mathcal{H} = \frac{1}{2}(\pi_\phi - eA_1)^2 + \frac{1}{2}\pi_1^2 + \frac{1}{2}\phi'^2 - \frac{ag^2}{2}A_1^2 - \frac{1}{2ae^2}(\pi_1' + e\phi')^2. \tag{9}$$

The first three terms are clearly positive. The last two terms also are positive if $a \leq 0$. On the other hand if this condition is violated, these terms become negative and can be made so large in magnitude that the positive terms cannot compensate for them. Thus the Hamiltonian can be bounded below only when $a < 0$. The present case is restricted to $a \neq 0$, but we have seen earlier that a positive Hamiltonian emerges also for $a = 0$, so that we can say that the condition for the Hamiltonian to be bounded below is indeed $a \leq 0$.

The first order equations of motion for the fields can be found from the Hamiltonian given by (9):

$$\dot{\phi} = \pi_\phi - eA_1, \tag{10}$$

$$\dot{A}_1 = \pi_1 - \frac{1}{e^2 a}\pi_1'' + \frac{1}{ea}\phi'', \tag{11}$$

$$\dot{\pi}_\phi = (a^{-1} - 1)\phi'' + \frac{1}{ea}\pi_1'', \tag{12}$$

$$\dot{\pi}_1 = e\pi_\phi - (1-a)e^2 A_1. \tag{13}$$

These lead to the simple second order equations

$$(\Box + (1-a)e^2)\pi_1 = 0. \tag{14}$$

$$\Box[\pi_1 - (1-a)e\phi] = 0, \tag{15}$$

These show that $\pi_1 + (1-a)e\phi$ represents a massless free field and π_1 a massive free field with mass $\sqrt{(1-a)e^2}$. In other words, there is a massive particle, as in the exceptional situation, but now there is a massless particle too. The massive particle can be regarded as a gauge boson which has acquired mass or a bound state of a fermion and an antifermion as before. The theory of a massless boson in two dimensions contains fermionic excitations, so that there is also a massless fermion in the spectrum now. Indeed, the massless boson can be regarded as the bosonized form of this massless fermion, which can be explicitly constructed if desired by standard bosonization formulas. This suggests that there is no confinement in this scheme.

3 Anomaly Cancelation Based on Dirac's Formalism

The variant of the Schwinger model described in Sec. 1 has gauge anomalies. So the model is described by a set of second class constraints. In this section this theory with second class constraints has been converted into a theory with first classs constraints following [7] formalism. In this new variant there is two second class constraints

$$\omega_1 = \pi_0, \tag{16}$$

$$\omega_2 = \pi_1' - e\phi' - ae^2 A_0. \tag{17}$$

and they satisfy the commutation relation

$$C = [\omega_1(x), \omega_2(y)] = ae^2 \times \begin{pmatrix} 0 & \delta(x-y) \\ -\delta(x-y) & 0 \end{pmatrix}. \tag{18}$$

Now two fields θ_1 and θ_2 are introduced to extend the phase space satisfying the relation

$$C^{-1}(x,y) = [\theta_1(x), \theta_2(y)]. \tag{19}$$

It is easy to show that the second class constraints (16) and (17) turns inti first class constraints if a new pair of canonical fields θ and π_θ are introduced. the fields θ_1 and θ_2 will be defined by the above pair of canonical fields later on. The second class constraints turns into first class constraints in the following way

$$\tilde{\omega}_1 = \omega_1 - e\theta, \tag{20}$$

$$\tilde{\omega}_2 = \omega_2 + ea\pi_\theta. \tag{21}$$

The primary Hamiltonian is

$$\mathcal{H}_p = \mathcal{H} + \pi_0 v_0, \tag{22}$$

where $v_0 = A_1'$.

The Hamiltonian with these first class set of constraints will be of the form

$$\tilde{H} = \int H + \frac{1}{2}\int [dxdy\theta_1(x)M_{11}(x,y)\theta_1(y) + \theta_2(x)M_{22}(x,y)\theta_2(y)] \tag{23}$$

where $\theta_1 = \frac{\pi_\theta}{e}, \theta_2 = \frac{1}{ea}\theta, M_{11} = -ae^2\delta(x-y), M_{22} = ae^2\delta(x-y)$.

So the first class Hamiltonian coming out to be

$$\tilde{\mathcal{H}} = \mathcal{H} - \frac{1}{2}ae^2(\pi_\theta^2 + \frac{1}{2}\theta'^2). \tag{24}$$

It is the gauge invariant Hamiltonian corresponding to the Lagrangian (3). The symmetry is confirmed from the first class nature of the constraints (20, 21). The physical containts does not change because the auxiliary fields just extend the phase space and allocate themselvs in the unphysical sector of the theory.

The effective action corresponding to the Hamiltonian(24) is

$$S_{eff} = \int dx[\pi_0\dot{A}_0 + \pi_1\dot{A}_1 + \pi_\phi\dot{\phi} + \pi_\theta\dot{\theta} - \tilde{\mathcal{H}}]. \tag{25}$$

If the momenta are redefined by

$$\pi_1 = \dot{A}_1 - A_0', \tag{26}$$

$$\pi_0 = e\theta, \tag{27}$$

$$\pi_\phi = \dot{\phi} - eA_1, \tag{28}$$

$$\pi_\theta = \dot{\theta} - \theta'. \tag{29}$$

It can be shown that the effective action (25) reduces to

$$\begin{aligned} S_{eff} &= \int dx[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e\epsilon_{\mu\nu}\partial^\mu\phi A^\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}ae^2A_\mu A^\mu \\ &- \frac{1}{2}a((\partial_\mu\eta)(\partial^\mu\eta)) + e\partial_\mu\eta A^\mu, \end{aligned} \tag{30}$$

where $\eta = a\theta$.

4 Gauge Invariant Reformulation without extending the phase space

The idea was first developed by Mitra and Rajaraman([8, 9]. The method based on the constrainet structure of the theory. Depending on the constrint structure one can get different gauge invariant action. Obviously, the physical containts are same in all the geuge invariant reformulants. Their methods are applicable in both the single chain and multi chain constrained systems. The main idea is to reduce the half of the constraints from a second class constrained system keeping the first class constraints only. This new generalized Schwinger model contains only two second class constraints and there is only one possiblity to make it gauge invariant which is to eleminate the constraint ω_2 keeping ω_1 as usual i.e., to change the Hamiltonian in such a way that $\dot{\omega}_1 = 0$ is satisfied. The Hamiltonian will be of the form

$$\tilde{\mathcal{H}} = \mathcal{H} + \frac{d}{e^2}(\pi_1' - e\phi' - e^2A_0)^2. \tag{31}$$

The codition $\dot{\omega}_1 = 0$ requires $d = \frac{1}{a}$.

The new equations of motion corresponding to the new Hamiltonian $\tilde{\mathcal{H}}$ are

$$\dot{\phi} = \pi_\phi - eA_1, \tag{32}$$

$$\dot{A}_0 = v_0, \tag{33}$$

$$\dot{A}_1 = \pi_1 - A_0' - \frac{1}{e^2 a}(\pi_1' - e\phi' - ae^2 A_0)'. \tag{34}$$

The first order Lagrangian is

$$\begin{aligned} \tilde{\mathcal{L}} &= \frac{1}{2}(\dot{\phi}^2 - \phi'^2) + e(A_1\dot{\phi} - A_0\phi') - \frac{1}{2}ae^2(A_0^2 - A_1^2) \\ &+ \frac{1}{a}(\phi' + eA_0)^2 + \frac{1}{2}\pi_1(\dot{A}_1 + \frac{1}{ae}\phi'') \end{aligned} \tag{35}$$

The gauge transformation generator is $\lambda \int dx \pi_0$, where λ is a c-number which trans form A_0 as $-\lambda$ and the other fields remain unchanged. It can be shown easily that the total change in the Lagrangian due to the above transformation $\Delta\tilde{\mathcal{L}} = 0$. So the Lagrangian (35) is gauge indariant.

5 BRST Invariant Reformulation

The BRST tecnique is to enlarge the Hilbart space of a gauge theory and to replace the notation of gauge transformation which shifts the operator by C-number function with BRST transformation which mixes operator having different statistics. It is very effective when one tries to discuss the renormalization property of a theory. One generaly exploit the BRST symmetry instead of exploiting the original gauge symmetry.

The combined formalism of Batalin ,Fradkin and Vilkovisky [2, 3, 15] for quantization of a system is based on the idea that a system with second class constraint can be made effectively first class in the extended phase space. The field needed for this conversion turns out to the Wess-Zumino scalar with the proper choise of gauge condition, as pointed out by FIK.

If a canonical Hamiltonian in terms of the canonical pairs $(p_i, q^i), i = 1, 2.......N$ is considered subjected to a set of constraints $\Omega_a = 0, a = 1, 2.......n$ and it is assumed that the constraints satisfy the following algebra.

$$[\Omega_a, \Omega_b] = i\omega_c U_{ab}^c, \tag{36}$$

$$[H_c, \Omega_a] = i\Omega_b V_c^b, \tag{37}$$

then m no of additional condition $\Phi_a = 0$ with $det[\Phi_a, \Omega_b] \neq 0$ have to be imposed inorder to single out the physical degrees of freedom.

The Hamiltonian consistent with the set of constraints $\Omega_a = 0$ and $\Phi_a = 0$ together with Hamiltonian equation of motion is obtained from the action

$$S = \int dt[p_i q^i - H_c(p_i, q^i) - \lambda^a \Omega_a + \pi_a \Phi^a], \tag{38}$$

where λ^a and π_a are lagrrangian multiplier field satisfying the relation $[\lambda^a, \pi_b] = i\delta_b^a$.

Now introducing one pair of canonical ghost field $(C^a, \bar{P}_a)$ and one pair of canonical antighost field $(P^a, \bar{C}_a)$ for each pair of constraints an equivalance can be made to the initial theory of the unconstrained phase space. So the quantum theory can be described by the action

$$S_{qf} = \int dt[p_i q^i + \pi^a \dot{\lambda}_a + \bar{P}^a \dot{C}_a + +\bar{C}^a \dot{P}_a - H_{BRST} + i[Q, \psi]. \tag{39}$$

H_{BRST} is the minimal Hamiltonian as termed by Batalin and fradkin, given by

$$H_{BRST} = H_c + \bar{P}_a V_b^a C^b. \tag{40}$$

The BRST charge and the fermionic gauge fixing function are given by

$$Q = C^a \omega_a - \frac{1}{2} C^b C_c U_{ab}^c - P^a \pi_a, \tag{41}$$

$$\psi = \bar{C}_c \chi^a + \bar{P}^a \lambda^a, \tag{42}$$

where χ_a are given by the gauge fixing condition $\Phi_a = \dot{\lambda}_a + \chi_a$.

The model consider here is described by the Lagrangian (3), The canonical Hamiltonian and the momenta are given in (6),(3), (4) and(5). The two second class constraints are

$$\omega_1 = \pi_0, \tag{43}$$

$$\omega_2 = \pi_1' - e\phi' - ae^2 A_0. \tag{44}$$

Now introducing BF field θ and π_θ the constraints are made first class

$$\tilde{\omega}_1 = \omega_1 - e\theta, \tag{45}$$

$$\tilde{\omega}_2 = \omega_2 + ea\pi_\theta. \qquad (46)$$

The time involution of the constraints are

$$\dot{\omega}_1 = i\omega_2, \qquad (47)$$

$$\dot{\omega}_2 = i\omega''. \qquad (48)$$

In general the first class Hamiltonian consistent consistent with the constraints $\tilde{\omega}_1$ and $\tilde{\omega}_2$ will be the original Hamiltonian added with a polynomial of BF field. And the polynomial will be determined by the condition that the new first class constraints will satisfy the same time involution like the old second class set of constraint (47) and (48)The extra term H_{BF} is found out to be

$$H_{BF} = -\frac{1}{2}(\pi_\theta^2 + \frac{1}{a^2}\theta'^2). \qquad (49)$$

The BRST charge and the fermionic gauge fixing condition are

$$Q = \int dx[B_1 P^1 + B_2 P^2 + C_1 \tilde{\omega}^1 + C_2 \tilde{\omega}^2], \qquad (50)$$

$$\psi = \int dx[\bar{C}_1 \chi^1 + \bar{C}_2 \chi^2 + \bar{P}_1 N^1 + \bar{P}_2 N^2]. \qquad (51)$$

Here the gauge fixing function are chosen to be $\chi_1 = A_0$ and $\chi_2 = \partial^1 A_1 + \frac{\alpha}{2} B_2$. It is easy to check that

$$[Q, H_{BRST}] = 0 \qquad (52)$$

$$[[\psi, Q], Q] = 0 \qquad (53)$$

$$Q^2 = [Q, Q] = 0 \qquad (54)$$

The effective action is given by

$$\begin{aligned} S_{eff} &= \int dx[\pi^0 \dot{A}_0 + \pi^1 \dot{A}_1 + \pi_\phi \dot{\phi} + \pi_t heta\dot{\theta} + B_2 \dot{N}^2 \\ &+ \bar{P}_1 \dot{C}^1 + \bar{P}_2 \dot{C}^2 + \bar{C}_2 \dot{P}^2 - H_{tot}, \end{aligned} \qquad (55)$$

where $H_{tot} = H_{BRST} - i[Q, \psi]$.

Here the term $\int dx[B_1\dot{N}^1 + \bar{C}_1\dot{P}^1 = i[Q, \int dx B_1\dot{N}^1]$ is suppressed in the Legender transformation by replacing χ_1 with $\chi_1 + \dot{N}_1$. The generating function is now given by

$$Z = \int [DM] exp(iS_{eff}), \tag{56}$$

where

$$\begin{aligned} [DM] &= [DA_0 D\pi^0][DA_1 D\pi^1][D\phi D\pi_\phi][D\theta D\pi_\theta] \\ &\times [DC^1 D\bar{P}_1][DC^2 D\bar{P}_2][DP^1 D\bar{C}_1] \\ &\times [DP^2 D\bar{C}_2][DN^1 DB_1][DN^2 DB_2] \end{aligned} \tag{57}$$

In order to derive the covariant action one should eliminate N_1, B_1, C^1, $\bar{C}_1$, P^1, $\bar{P}_1$, A_0 and π_0 by Gaussian integration.After integration the effection reduces to

$$\begin{aligned} S_{eff} &= \int dx[\pi_0\dot{A}_0 + \pi_1\dot{A}_1 + \pi_\phi\dot{\phi} + \pi_\theta\dot{\theta} + B\dot{N} + \bar{P}\dot{C} + \bar{C}\dot{P} \\ &- [\frac{1}{2}(\pi_1^2 + \pi_\phi^2 + \phi'^2 - eA_1\pi_\phi - \frac{1}{2}(a-1)e^2A_1^2] - ea\theta' \\ &+ \frac{a}{2}(\pi_\theta^2 + \frac{1}{a^2}\theta'^2) + B(A_1' + \frac{a}{2}B) \\ &+ N(\pi_1' - e\phi' + ea\pi_\theta) - C\bar{C}''. \end{aligned} \tag{58}$$

Again integrating over $\pi_1, \pi_\phi, \pi_\theta$ and $\bar{P}$ one can obtain

$$\begin{aligned} S_{eff} &= \int dx[\frac{1}{2}(\partial_\mu\phi)(\partial^\mu\phi) + e\epsilon_{\mu\nu}\partial^\mu\phi A^\nu - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}ae^2A_\mu A^\mu \\ &- \frac{1}{2}a((\partial_\mu\eta)(\partial^\mu\eta)) + e\partial_\mu\eta A^\mu \\ &- \partial_\mu\bar{C}\partial^\mu C + A_\mu\partial^\mu B - \frac{\alpha}{2}B^2. \end{aligned} \tag{59}$$

where N is replaced by $-A_0$ and η is defined by $\eta = a\theta$. The action is invariant under the BRST transformation $\delta X = \lambda \int dx[Q, X]$ where X stands for the field variables of the above action and λ is a grassmanian parameter.

6 Conclusion

To summarize, we have looked at the familiar Schwinger model in the one parameter class of regularizations used in studies on the chiral Schwinger model and shown that although only global gauge invariance is maintained, the theory is sensible in every way and can be solved exactly. The spectrum consists of the usual massive boson with the mass explicitly depending on the regularization parameter – but there is also a massless fermion, which is the main difference from the usual treatment.

Gauge invariant reformulation is done in two ways one in the extended phase spaces and the other in the original phase space both the cases the gauge invariant action is superficially different from the actual action but the physical containts are alike. The external field introduced to enlarge the phase space is shown to be identical to the Wess-Zumino scalar when Woutzasec's formalism is considered. But in case of Mitra and Rajaraman' s formalism a gauge invariant action is obtained without the emergence of any Wess-Zumino term.

Following BVF formalism a BRST invariant action is obtained. In the usual Hamiltonian formalism of a gauge invariant theory one some times need to destroy the gauge symmetry under the introduction of some gauge fixing terms. However, BRST invariant Hamiltonian which has beeen reformulated will help one to work in an extended phase space on which only a subspace corresponds to the state of physical interest.

The Schwinger model is well known to be solvable, the new generalization differs from the usual one by the mass terms of the gauge fields do not loose the solvablity. A gauge invariant as well as a BRST invariant reformulation is made so that calculations ordinarily made with the gauge invariant Schwinger model can be repeted here. New physical consequences will arise, as is indicated by the fact that even the spectrum gets altered.

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