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N=1 Dual String Pairs and Gaugino Condensation

Shamit Kachru*
Lyman Laboratory of Physics
Harvard University
Cambridge, MA 02138

Eva Silverstein†
Joseph Henry Laboratories
Jadwin Hall
Princeton University
Princeton, NJ 08544

We study a class of four-dimensional N=1 heterotic string theories which have non-trivial quantum dynamics arising from asymptotically free gauge groups. These models are obtained by orbifolding 4d N=2 heterotic/type II dual pairs by symmetries which leave unbroken products of nonabelian gauge groups (without charged matter) in a "hidden sector" on the heterotic side. Such models are expected to break supersymmetry through gaugino condensation in the hidden sector. We find a dual description of the effects of gaugino condensation on the type II side, where the corresponding superpotential arises at tree level. We speculate that the conformal field theory underlying the type II description may be related to a class of geometrical nonsupersymmetric string compactifications.

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* kachru@string.harvard.edu
† silver@puhep1.princeton.edu

1. Introduction

There has been much progress in recent months in unifying various string theories in different dimensions using the appropriate versions of strong/weak coupling duality. In compactifications with enough supersymmetry, the low-energy physics is more or less determined by the constraints of supersymmetry. In compactifications with the equivalent of $N \leq 2$ supersymmetry in four dimensions, there emerges the possibility of nontrivial dynamics in the infrared. While the most famous example of "string-string" duality between heterotic and type II strings involves theories with the equivalent of N=4 supersymmetry in four dimensions [1][2], more recently dual heterotic/type II pairs with $d = 4, N = 2$ supersymmetry have been discovered [3][4] and studied in some detail [5,6,7,8,9,10]. One finds that the dual description renders instanton effects computable using classical string theory.

The theories of interest for describing low-energy particle physics have $N \leq 1$ supersymmetry in four dimensions. In particular, there has been much interest in $4d$ $N = 1$ heterotic models for string phenomenology. One mechanism for supersymmetry breaking that has been much studied in this context is gaugino condensation in a hidden sector [11,12,13,14,15,16,17].

Starting from the $N = 2$ dual pairs, one can form $N = 1$ dual pairs by freely acting orbifolds [8][18]. Unfortunately, the N=1 pairs constructed to date have had trivial dynamics in the infrared. There are several different classes of infrared dynamics, parametrized by the spectrum of gauge fields and charged matter present in the ultraviolet, that have been studied fruitfully in supersymmetric field theory (see [19] for a review). Understanding the quantum behavior of N=1 string vacua in four dimensions will involve learning how string theory recovers and generalizes these phenomena.

In this paper we will study the role of duality in elucidating the effects of gaugino condensation for a class of examples. Specifically, it is possible to construct freely acting orbifolds producing N=1 dual pairs which on the heterotic side have pure factors in the gauge group. Of particular interest are models with more than one simple factor in the pure gauge group, as some of these models are expected to lead to stable vacua with broken supersymmetry [13][14][15]. We describe how to construct such models in §2.

On the type II side, this appears mysterious at first sight. The singularities of the vector multiplet moduli space of the $N = 2$ theory do not lead to nonabelian gauge symmetry enhancement (except in the case of nonasymptotically free theories [20]). Therefore, in the type II $N = 1$ orbifold, there is no obvious origin of quantum supersymmetry breaking effects. This suggests that the supersymmetry breaking, if present, should be evident classically. Said differently, despite the lack of nonrenormalization theorems to guarantee the utility of strong/weak coupling duality in $N = 1$ string theory, the absence of nonabelian dynamics on the type II side essentially requires its description of the physics to be perturbative, leading to a useful duality. We will propose a tree level mechanism on the type II side which reproduces the expected potential generated by gaugino condensation in §3.

The mechanism can be summarized as follows. In the global limit it reduces to that which Seiberg and Witten used for understanding the mass gap of pure $N = 1$ gauge theory by perturbing pure $N = 2$ gauge theory with a mass term for the adjoint scalar ϕ [21]. In their scenario turning on a mass perturbation leads to a vacuum expectation values for the light monopole fields present at the special singularities of the N=2 moduli space. In string theory it is not possible to turn on masses by hand. We find nevertheless that the orbifold[1] spectrum contains a massive field, which has the same global couplings as the ϕ field studied in [21], and which becomes light as the heterotic coupling becomes weak. In §3 we explain how we can infer the presence and couplings of such fields. We then study the bosonic potential in the resulting low-energy supergravity theory and compare to the expectations of gaugino condensation on the heterotic side. In §4 we recap and discuss directions for further exploration of these models.

The purpose of this paper is to demonstrate how the type II side manages to encode perturbatively the physics of the heterotic side. We wish to note here that the utility of the proposed duality comes from the opposite approach. The perturbative physics of the type II side should contain a wealth of information about the quantum behavior of the heterotic side, and is a promising starting point for a systematic study of the details of such models. There has been other recent work on supersymmetric/nonsupersymmetric duality in field theory [22] and string theory [23][24]. Earlier attempts to use strong/weak coupling duality to shed light on gaugino condensates (by assuming an $SL(2, Z)$ S-duality acting on the dilaton-axion multiplet) can be found in [25].

[1] Actually on the type IIA side we will be discussing an orientifold.

2. Construction of Dynamical Duals

Our starting point is the adiabatic prescription introduced by Vafa and Witten for producing dual string pairs from known examples of string-string duality. The $N = 2$ compactifications have the structure of $K3$ fibrations ($\mathbf{CP}^1$,$K3$) on the type II side [7] and T^4 fibrations ($\mathbf{CP}^1$,T^4) on the heterotic side [8]. Act with a freely acting symmetry (in our case $(z_1, z_2) \to (\bar{z}_2, -\bar{z}_1)$) on the base and approach the large radius limit of the $\mathbf{CP}^1$. Then the local observer sees the physics of a compactification on $K3$ or T^4 (on the type II and heterotic sides, respectively) and maps reliably to the dual theory using the well-established $6d$ string-string duality [1][2].

We will now discuss the class of orbifolds of interest to us, first in the heterotic and then in the type II description.

2.1. The Heterotic side

In order to apply the adiabatic argument of [8], we look for freely acting symmetries with which to orbifold the $N = 2$ examples of [3]. On the heterotic $K3 \times T^2$, we can use the Z_2 given by the Enriques involution on the $K3 \sim (\mathbf{CP}^1, T^2)$ and the reflection -1 on the T^2 [4][8].

Of course, we must also choose an embedding of the Z_2 in the gauge degrees of freedom. This will determine the surviving gauge group and charged matter content. We can obtain $N = 1$ models with pure factors in the gauge group as follows, starting from models 6-8 in §4.5 of [3]. For definiteness, consider model 7, obtained on the heterotic side by embedding an $SU(2)$ bundle with $c_2 = 20$ in one of the two E_8 factors (which we will call $E_8^{\rm obs}$), breaking it to E_7, and embedding a rank two bundle with c_2=4 into an enhanced $SU(2)$ arising from the T^2 fixed at $\tau = \rho$. The generic spectrum of 11 vectors and 377 hypermultiplets is obtained as the Higgs phase of the E_7 and the Coulomb phase of the second E_8 (which we will call E_8^H). This model has a known type II Calabi-Yau dual described in [3].

If we consider the heterotic side *before* Higgsing the E_7, then we see that any further orbifolding to get an $N = 1$ model with modular-invariant embedding of the orbifold group only into $E_8^{\rm obs}$ will produce a model with a hidden sector. Here we should emphasize that although the $N = 2$ dual pair of [3] involves the heterotic model in which the E_7^{obs} is completely Higgsed, one should be able to follow both sides of the duality through appropriate "extremal transitions" to the model with the E_7 unbroken (examples of such transitions have been discussed in detail in [26][9][10][27]). In general there is no guarantee that the appropriate type II dual will still be a Calabi-Yau compactification, but for our example there *is* in fact a good candidate for the "unHiggsed" dual, as we will explain presently.

The model we are discussing has a generic spectrum – before Higgsing the observable E_7 – of 62 hypermultiplets and 18 vector multiplets. This would correspond to a type IIA string compactification on a Calabi-Yau with Hodge numbers $h_{11} = 17, h_{21} = 61$. There is indeed a known $K3$ fibration with these Hodge numbers, given by the hypersurface of degree 68 in $\mathbf{WP}^4_{3,3,8,20,34}$ [28]. The type IIA compactification on this manifold is a good candidate for the dual of our "unHiggsed" heterotic theory.

Now that we have a candidate dual for the heterotic theory with $E_8 \times E_7 \times U(1)^3$ gauge group (and with 8 **56**s of E_7) we need to decide which sorts of free group actions we want to orbifold by to obtain interesting N=1 models. Of more interest for phenomenology than the models with E_8^H unbroken are models with several pure nonabelian factors in the gauge group [13]. We can obtain such models by first turning on discrete Wilson lines (consistent with the above Z_2 action on the T^2) to break E_8^H to a subgroup and then orbifolding by the Z_2.[2] Equivalently, this can be described as embedding translations generating the T^2, which are part of the space group of the orbifold, into E_8^H. This procedure is constrained by level-matching and by the relations of the space group [29].

There are several consistent choices which produce product hidden sector groups. For example, one choice that leads to a hidden sector gauge group $G^H = SO(8)^2$ is obtained as follows. Take Wilson lines $A_1 = L_1/2$ and $A_2 = L_2/2$ around the two cycles of the T^2, where $L_1 = (0,0,0,0,1,1,1,1)$ and $L_2 = (-2,0,0,0,0,0,0,0)$ are vectors in the E_8 root lattice. Since A_1^2, A_2^2, and $A_1 \cdot A_2$ are integers, this satisfies level-matching. With Wilson lines turned on, the momentum lattice becomes

$$p_L = (P^I - A_i^I n^i, G^{ij}(\frac{m_j}{2} + \frac{A_j^I}{2}P^I - B^{ij}n_j - \frac{A_i^I A_j^I}{4}n^j) + n^i) \tag{2.1}$$

$$p_R = G^{ij}(\frac{m_j}{2} + \frac{A_j^I}{2}P^I - B^{ij}n_j - \frac{A_i^I A_j^I}{4}n^j) - n^i \tag{2.2}$$

[2] In turning on Wilson lines in model 7 of [3], we simultaneously move away from $\tau = \rho$ in such a way as to preserve, in the presence of the Wilson lines, the enhanced $SU(2)$ in which we have embedded a $c_2 = 4$ $SU(2)$ bundle

This shows that the surviving gauge bosons have root vectors satisfying $A_i^I P^I \in \mathbf{Z}$. The E_8 root lattice consists of vectors of the form $\pm e_i \pm e_j$ and $\frac{1}{2}(\pm e_1 \pm e_2 \ldots \pm e_8)$. All of the first set and none of the second set satisfy $A_2 \cdot P \in \mathbf{Z}$. There are 48 vectors in the first set which satisfy $A_1 \cdot P \in \mathbf{Z}$. These split up into two copies of the root lattice of $SO(8)$. Combined with the 8 Cartan generators, this gives the dimension 56, rank 8 group $G^H = SO(8) \times SO(8)$. Other hidden sector product groups can be obtained similarly: The discussion of the type II side below applies to all models of this type.

2.2. The Type IIA side

The heterotic orbifold was constructed by a Z_2 which acted freely on the base ($\mathbf{CP}^1$) of the elliptic fibration as follows:

$$(z_1, z_2) \to (\bar{z}_2, -\bar{z}_1). \tag{2.3}$$

As we mentioned in §2.1, the $N = 2$ heterotic model has a proposed dual, type IIA string theory compactified on the Calabi-Yau hypersurface in $\mathbf{WP}^4_{3,3,8,20,34}$ [27][28]. This manifold is a $K3$ fibration, with the $K3$ fiber being given by a degree 34 hypersurface in $\mathbf{WP}^3_{3,4,10,17}$.

By the adiabatic argument, we expect a type II dual of the $N = 1$ heterotic orbifold model which is obtained by translating the action of the Z_2 on the heterotic coordinates to an action on the harmonic forms of the $K3$ fiber [2][8]. On the heterotic side the orbifold left us with a pure gauge factor, projecting out the charged fields which in the N=2 theory parameterized the Coulomb phase of E_8^H. The singularities corresponding to enhanced gauge symmetry on the heterotic side map to the conifold singularities of the Calabi-Yau moduli space on the type II side [3][4]. The corresponding orbifold on the type IIA side will project out the scalars, $a^i_{b,D}$ $i = 1, \ldots, \mathrm{rank}(G^H_b)$. These are the moduli which could move us away from the conifold locus dual to the enhanced hidden gauge symmetry locus of the heterotic string side. On the other hand, the abelian vectors in $N = 2$ vector multiplets will survive the orbifold projection.

Before embarking on an analysis of the physics of the dual descriptions, we should make sure that the heterotic orbifold indeed maps to a bona fide orbifold on the type II side. That is, does the action G on the harmonic forms of $K3$ implied by the action on the Narain lattice on the heterotic side [2] determine a symmetry of the $K3$ itself? More precisely, we must ensure that there is a symmetry of the worldsheet action on the type II side by which we can orbifold. The action on the base (2.3) indicates that the symmetry is antiholomorphic and orientation reversing which means that we must also exchange left and right movers on the worldsheet, giving us an orientifold as in the examples of [8]. The novelty in our construction is that we must also find a symmetry which freezes the $K3$ fibers at a singular $K3$, dual to the enhanced gauge symmetry present in the heterotic orbifold [1][30].

If the heterotic theory is developing an enhanced gauge symmetry group G, then the dual description involves a $K3$ in which curves C_i ($i = 1, \cdots, \mathrm{rank(G)}$) are shrinking to zero size (and the associated worldsheet θ angles are also vanishing [30]). In general, given a smooth rational curve C, it satisfies $C \cdot C = -2$ (where $\cdot$ denotes the intersection product between homology classes), so one can consider an automorphism of $H^2(K3)$ given by the "reflection"

$$X \to X + (X \cdot C)C \tag{2.4}$$

which takes $C \to -C$. This automorphism is *not* associated with a symmetry of the $K3$ – to explain what it *is* associated with, we need to recall and extend the notion of a birational transformation. [3]

Our extended notion of birational transformation will be as follows: A birational transformation between X and Y, both of dimension d, is an algebraic cycle $Z \subset X \times Y$, also of dimension d, such that for appropriate dense open subsets $U \subset X$ and $V \subset Y$ the intersection of Z with $U \times V$ is the graph of an isomorphism. Any such cycle induces a map $H^k(X) \to H^k(Y)$ in a natural way. The important point for us, however, is that Z is allowed to contain more than one component: All but one of the components will map to proper subvarieties in both X and Y, and so will be disjoint from $U \times V$.

In intuitive terms, this extension of the notion of birational transformation has the following effect. As far as complex structures are concerned, any birational spaces X, Y differ only at complex co-dimension two. That is, one can extend the isomorphism given by $U \times V$ to hold up to codimension two, in a manner consistent with the complex structures of X and Y. However, in string theory, we are also interested in keeping track of the Kahler classes, and these can obstruct the extension of the isomorphism even to codimension one subspaces of X and Y.

So in fact, (2.4) is the action on the cohomology induced by a birational transformation in the sense discussed above. More generally, one can define such a "reflection" associated

[3] We thank D. Morrison for very helpful discussions about this and the following.

with any set of rational curves C_i (see the discussion in §3 of [31] and also in [32]). These reflections are also associated with birational transformations (in this extended sense) between distinct $K3$s. The "extra" components of Z will be $CP^1 \times CP^1$s given by the curves C_i in the two $K3$s. It turns out that the string compactifications on the two $K3$s are isomorphic, however, so these "reflections" generate Z_2s acting on the Teichmuller space for the moduli space of string theories on $K3$. Heuristically speaking, one should envision a cone divided into two "mirror image" cones A and B by a dividing wall in the center. A is the Kahler cone for one $K3$, and the $K3$ obtained by doing the reflection (2.4) (and the associated birational transformation) instead has Kahler cone B. The wall dividing the two cones A, B is the wall where the C_i shrink to zero area.

It is well known that at the *fixed point* of a symmetry group g which acts on the Teichmuller space, one obtains a conformal theory with an enhanced g symmetry. So at the fixed point of the Z_2 generated by such a reflection, one will find a conformal theory on $K3$ with an extra Z_2 symmetry g. It is precisely this Z_2 symmetry g that we must orbifold by, to freeze the $K3$ at the enhanced gauge symmetry point.

In the context of the main example we have been discussing, the C_i are the curves which shrink to zero size as the heterotic string develops its $G^H \times E_7$ gauge symmetry. We should orbifold the type IIA side by a combination of orientifolding and simultaneous action with g, to construct the dual to the heterotic theory.

What does g correspond to on the world sheet? One way to study this limit is by making use of a linear sigma model description of the model, following [33][34]. In this formalism, the Kahler modes corresponding to the sizes of the C_i are represented on the worldsheet by $U(1)$ gauge multiplets, with the coefficients $\vec{r}_i = (r_i^0, r_i^1, r_i^2)$ of (generalized) worldsheet Fayet-Iliopoulos D-terms giving the sizes of the C_i. Each gauge multiplet Σ_i contains four scalars σ_i which couple to charged hypermultiplets ϕ_α^i, $\tilde{\phi}_\alpha^i$. The bosonic potential is [34]

$$\begin{aligned} V =&\frac{1}{2e^2}\sum_i \Big\{ \Big([\sum_\alpha Q_i^\alpha (|\phi_\alpha^i|^2 - |\tilde{\phi}_\alpha^i|^2)] - r_i^0 \Big)^2 \\ &+ \Big(Re(\sum_\alpha \phi_\alpha^i \tilde{\phi}_\alpha^i) - r_i^1 \Big)^2 + \Big(Im(\sum_\alpha \phi_\alpha^i \tilde{\phi}_\alpha^i) - r_i^2 \Big)^2 \Big\} \\ &+ \frac{1}{2}\sum_i [\sum_\alpha Q_i^{\alpha\ 2} (|\phi_\alpha^i|^2 + |\tilde{\phi}_\alpha^i|^2)] |\sigma_i|^2 \end{aligned} \tag{2.5}$$

We see from (2.5) that for $r \to 0$ (with in addition the worldsheet θ angles set to zero), the model develops a singularity arising from the region of field space where $\sigma_i \to \infty$ and $\phi^i_\alpha = 0$. Indeed, this is the only regime where the model can be reliably studied semiclassically for $r_i = \theta_i = 0$. In this limit, the action $\Sigma \to -\Sigma$ on the worldsheet gauge multiplet becomes a symmetry of the worldsheet model (combined with $\phi \leftrightarrow -\tilde{\phi}$, though on the σ branch all the fields which couple to Σ are hugely massive). This appears to provide a worldsheet description of the geometrical action g described above. Moreover, the vertex operators for the Kahler blow-up modes are given by the fermions in the gauge multiplet [35], so this action indeed projects out the Kahler deformations corresponding to the C_i. As we will see below, the type II description of the heterotic dynamics that we will present relies only on the details of the action on the moduli and the action (2.3) on the base of the $K3$ fibration.

3. Black Hole Condensation and Supersymmetry Breaking

Having seen in §2 that the adiabatic argument allows us to construct N=1 dual pairs with highly nontrivial dynamics expected on the heterotic side, we now turn to a brief analysis of the dual descriptions of the infrared physics.

3.1. Heterotic expectations

Given the presence of a pure gauge group G^H, our expectation is that gaugino condensation will occur. In a globally supersymmetric theory, this would lead to a mass gap and a discrete set of degenerate vacua [36]. The gaugino condensate is

$$\langle \chi^{(b)}_\alpha \chi^{(b)\alpha} \rangle \sim \Lambda_b^3 e^{i\gamma} \tag{3.1}$$

where the phase γ corresponds to a given discrete choice of vacuum. This will lead to a superpotential

$$W = \sum_b h_b \ \Lambda_b^3(S) \tag{3.2}$$

where

$$G^H = \Pi \ G^b \tag{3.3}$$

and where b indexes the various hidden groups. In (3.2) we have noted that in string theory the scale Λ at which a given factor in G^H becomes strong really depends on the dilaton S. The constants h_b include the phases γ^b corresponding to the discrete choices of vacua [12][13].

By analyzing the resulting bosonic potential (taking into account the dependence of the Kahler potential K and the superpotential W on all the chiral superfields in the low-energy theory at weak coupling), one can in principle determine whether a stable vacuum exists at weak coupling with broken supersymmetry and determine the resulting vacuum energy (see [14][16] for a review of various approaches to this problem). Our problem is to understand how this structure is reproduced by the type II dual, and to see whether the dual description provides any useful insights about the physics, at least in particular limits.

3.2. Type II description

Now we turn to the question of how the type II string can reproduce the effects of gaugino condensation evident from the heterotic analysis. There are several points we must take into account, which will be crucial to the physics on the type II side. First of all, the heterotic orbifold indicates the spectrum of the theory for weak heterotic coupling, which is mapped to large $\mathbf{CP^1}$ radius R on the type II side. The purported nonperturbative vacuum on the heterotic side is at small nonzero coupling, corresponding to large finite R. The orbifold acts freely on the $\mathbf{CP^1}$ base. We are therefore interested in the perturbative spectrum on the type II side as a function of large finite R. There are two features of the conifold locus at $R \to \infty$ which are crucial to understanding the type II physics:

1) The massless states in the $N = 2$ theory that are projected out by the Z_2 become invariant when given quantized internal momentum leading to masses $\sim 1/R^2$ as $R \to \infty$. This implies that the full $N = 2$ supersymmetry is restored as $R \to \infty$.
2) The low-energy theory for type II at the conifold locus contains massless solitonic states [37] in addition to the perturbative states obtained from the orbifold. One can see that these massless solitons survive the transition from the N=2 theory to the N=1 orientifold by examining the monodromies of the gauge coupling functions [8].

We will now take these points into account systematically. The orbifold on the type II side will have the same action $(z_1, z_2) \to (\bar{z}_2, -\bar{z}_1)$ on the base of the $K3$ fibration as it had on the base of the elliptic fibration on the heterotic side. This turns the $\mathbf{CP}^1$ into $\mathbf{RP}^2$, which has nontrivial fundamental group $\pi_1(\mathbf{RP}^2) = Z_2$. Therefore, a state that is projected out by the orbifold will have a massive version, with appropriate momentum along the nontrivial cycle γ, which is invariant under the Z_2. More explicitly, in the adiabatic limit we can send the original string state localized along γ with momentum $p = \frac{1}{R}$. Then if the original vertex operator V transforms as $V \to -V$ under the Z_2, the state

$$V' = e^{i\frac{x}{R}} f(x_\perp) V \tag{3.4}$$

will be invariant. In (3.4) x is the coordinate along the nontrivial cycle of the $\mathbf{RP}^2$, and $f(x_\perp)$ localizes the string along γ. The orbifold takes $x \to x + \pi R$ so that the momentum factor gives a compensating factor of -1 under the Z_2.

This means that we will have massive versions of all the fields $a^i_{b,D}$, with mass proportional to $1/R^2$. In addition, we will have the monopole hypermultiplets M^b_i and $\tilde{M}^b_i$, which survive the orbifold on the type II side [8]. Because $N = 2$ supersymmetry is restored as $R \to \infty$, the massive $a^i_{b,D}$ fields will couple to the surviving monopole fields as in the $N = 2$ theory.[4] As discussed in the work of Seiberg and Witten [21] (see also [39][40]), in the global limit the mass terms involve the gauge invariant global coordinates u^b_α (where α indexes the Casimirs of the Lie algebra) on the moduli space, which are functions of the $N = 2$ scalar superpartners of the gauge bosons, $a^i_{b,D}$, $i = 1, \ldots, \mathrm{rank}(G^H_b)$.

This reasoning tells us that our massive fields appear in the superpotential in terms proportional to the quadratic Casimirs u^b_2 of the hidden groups G^H_b. In the string theory context, the coupling constant is the heterotic dilaton field S, which maps by duality to the chiral superfield y containing the $\mathbf{CP^1}$ radius R. So in our case $u^b_\alpha = u^b_\alpha(a_{b,D}, y)$. Therefore as $R \to \infty$ there is a superpotential which looks like

$$W_{II} = \sum_b \left(m_b u^b_2(a^i_{b,D}, y) + \sum_{i=1}^{r(b)} M^b_i a^i_{b,D} \tilde{M}^b_i \right) \tag{3.5}$$

on the type II side. Here $r(b)$ is the rank of the bth factor in the hidden gauge group.

In analyzing the resulting bosonic potential, we will (i) reproduce the general structure of the potential arising from gaugino condensation on the heterotic side and (ii) discover that the monopole fields have vacuum expectation values, suggesting a geometric description by analogy with the conifold transitions that occur in the $N = 2$ context [41][26].

Before analyzing the physics of (3.5), it is helpful to remember the simplest case discussed in [21]. In order to recover the physics of the N=1 SU(2) theory from their solution of the N=2 SU(2) theory, Seiberg and Witten perturb the N=2 theory by a

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[4] This should also follow from the appropriate computation of the coupling of the vertex operator to the D-brane monopole [38].

superpotential which gives a mass to the adjoint scalar in the N=2 vector multiplet. Taken together with the couplings of the monopole fields $M, \tilde{M}$ which become massless at special points this implies

$$W = mU(a_D) + \sqrt{2} a_D M \tilde{M} \ . \tag{3.6}$$

Then from the equations of motion together with the condition of D-flatness they find that

$$|\langle M \rangle| = |\langle \tilde{M} \rangle| = (\frac{-mU'(0)}{\sqrt{2}})^{1/2} \neq 0 \ . \tag{3.7}$$

The monopoles condense and give a mass to the (dual) U(1) gauge field by the magnetic Higgs mechanism. The resulting low energy theory has a gap – this is the dual explanation of confinement by monopole condensation.

Our expectation in the context of heterotic/type II duality is that the superpotential (3.5) of our type II duals will give an analogous picture, with light black hole condensation providing the dual description of gaugino condensation and supersymmetry breaking. To make this more concrete, we must compute the bosonic potential

$$V = e^K \left(D_i W G^{i\bar{j}} D_{\bar{j}} W - 3|W|^2 \right) + \frac{1}{2} g^2 D^2 \tag{3.8}$$

in terms of the Kahler potential K and superpotential W.

Let us expand the superpotential (3.5) in $a^i_{b,D}$ in anticipation of finding a minimum at small $a^i_{b,D}$.

$$u_2(a^i_{b,D}, y) = e^{i\gamma^{(b)}} \Lambda_b^2(y) + \frac{\partial u_2}{\partial a^i_{b,D}} a^i_{b,D} + \dots \tag{3.9}$$

Recall also that the matching between the high and low-energy theories gives the relation

$$m_b \Lambda^2_{\text{b, high}} = \Lambda^3_{\text{b, low}} \tag{3.10}$$

so we obtain a superpotential

$$W_{II} = \sum_b \left(e^{i\gamma^{(b)}} \Lambda_b^3 + \frac{\partial u_2^b(y)}{\partial a^i_{b,D}} a^i_{b,D} + \sum_{i=1}^{r(b)} M_i^b a^i_{b,D} \tilde{M}_i^b \right) \tag{3.11}$$

Since we are interested in comparing to the heterotic side, let us work now in terms of the heterotic coupling S and consider Λ_b's dependence on S. Then we obtain the following bosonic potential

$$
\begin{aligned}
V =& e^K \Big[\sum_{b,i(b)} \big| h_b m_b u^b_{2,i}(S) + M^b_i \tilde{M}^b_i + K_i W \big|^2 \\
&+ \big| \sum_b [h_b \frac{\partial \Lambda^3_b(S)}{\partial S} + h_b \frac{\partial u^b_{2,i}(S)}{\partial S} a^i_{b,D}] + K_S W \big|^2 \\
&+ \sum_b \sum_{i,\bar{j}=1}^{r(b)} a^i_{b,D} M^b_i G^{\tilde{M}^b_i \bar{\tilde{M}}^b_j} \bar{a}^j_{b,D} \bar{M}^b_j + \sum_b \sum_{i,\bar{j}=1}^{r(b)} a^i_{b,D} \tilde{M}^b_i G^{M^b_i \bar{M}^b_j} \bar{a}^j_{b,D} \bar{\tilde{M}}^b_j \\
&+ \mathrm{F-terms\ of\ other\ fields} \\
&- 3 \Big| \sum_b \sum_{i=1}^{r(b)} \big(h_b \Lambda^3_b(S) + h_b u^b_{2,i}(S) a^i_{b,D} + M^b_i a^i_{b,D} \tilde{M}^b_i \big) + \mathrm{other\ fields} \Big|^2 \Big] \\
&+ \frac{1}{S_{II} + \bar{S}_{II}} \sum_b \sum_{i(b)=1}^{r(b)} \Big(|M^b_i|^2 - |\tilde{M}^b_i|^2 \Big)
\end{aligned}
\tag{3.12}
$$

where we have used the notation $\frac{\partial u^b_2}{\partial a^i_{b,D}} \equiv u^b_{2,i}$.

This expression of course depends on the Kahler potential. In general, one would expect the Kahler potential to receive significant loop (and nonperturbative) corrections, but duality allows us to work at arbitrarily weak coupling on the type II side–given the absence of field theoretic nonperturbative effects on the type II side–unless *stringy* nonperturbative effects fix the type II dilaton away from weak coupling. Assuming any such additional effects leave a minimum at weak type II coupling, the tree level Kahler potential should be a good approximation.[5] The vacuum will also depend on the contributions to the bosonic potential of fields other than those on which we are focusing. Different sets of assumptions and methods for analyzing the potential exist in the literature (see for example [14][16] and references therein).

Assuming there is a minimum near the $a^i_{b,D} = 0$ minimum of the rigid case (3.6), the monopole fields M and $\tilde{M}$ will minimize the first term in (3.12) (consistent with D-flatness) up to supergravity corrections. Setting $D_i W = 0$ yields

$$
\langle M^b_i \tilde{M}^b_i \rangle = -h_b m_b u^b_{2,i}(S) - K_i W \tag{3.13}
$$

[5] If this assumption is false, the duality we discuss still applies but the analysis of the bosonic potential changes accordingly to take into account its dependence on the moduli coming from $N = 2$ hypermultiplets.

Here the first term is in accord with (3.7) and the second term arises from supergravity. Now having integrated out the extra particles M, $\tilde{M}$, and a_D, we obtain the same general form of bosonic potential as arises from the heterotic side. We must then minimize with respect to the dilaton and all the other scalars in the model. Then supersymmetry is broken if there is any field ϕ for which

$$F_\phi = e^{\frac{K}{2}} G^{\phi\bar{\phi}} \left(\frac{\partial W}{\partial \phi} + \frac{\partial K}{\partial \phi} W \right) \neq 0 \tag{3.14}$$

in the vacuum.

As in the global situation analyzed in [21], the dual description of the effects of gaugino condensation involves a mass perturbation breaking the $N=2$ supersymmetry. We have seen that the orbifold produces the necessary massive mode as a Kaluza Klein excitation of the original variable u_2^b that was projected out.

One intriguing feature of the type IIA vacuum is the presence of nonzero vacuum expectation values of the monopole fields (wrapped two-branes) M and $\tilde{M}$. In the context of $N=2$ compactifications of the type II string theory, such vacuum expectation values can be turned on continuously when consistent with D- and F-flatness, giving transitions to other branches of the moduli space [26]. There is a well-known geometrical description of the conformal field theories involved in this process [41]. For example in type IIB string theory one approaches the conifold locus in complex structure moduli space by deformations causing appropriate S^3s to shrink to zero size. One can then resolve this singularity by replacing the tips of the resulting cones by $\mathbf{CP^1} \sim S^2$s.

At the generic conifold such a "small resolution" does not produce a Kahler manifold [42]. This was noted by Candelas, de la Ossa, Green, and Parkes, who speculated that such resolutions may correspond to supersymmetry-breaking directions [43]. The analysis presented here suggests a realization of these ideas through duality. By analogy with the quantum conifold transitions in the $N=2$ context, we expect that the nonzero monopole VEVs we have found correspond to a vacuum which has a conformal field theory description involving strings propagating on the non-Kahler resolutions of conifold singularities. If this analogy holds, then the fact that the corresponding conformal field theory is nonsupersymmetric might in fact provide the simplest method for establishing that supersymmetry is broken in such theories.

Given a conformal field theory description, a very general argument suggests that the leading approximation (in the type II coupling) to the cosmological constant *must* vanish in this class of theories. Although on the heterotic side detailed dynamical assumptions are usually invoked, on the type II side this statement follows simply from the fact that the leading contributions to the vacuum energy vanish by $SL(2,C)$ invariance.

4. Conclusions

We have seen that the construction of dynamical 4d N=1 dual pairs is possible by application of the adiabatic argument of [8]. In particular, one can realize "racetrack models" of supersymmetry breaking in a dual type II description.

Our analysis suggests that the tree-level superpotential on the type II side reproduces the bosonic potential expected from gaugino condensation on the heterotic side, by generalizing the mechanism of [21] which explained the gap of pure N=1 SU(2) gauge theory. A careful study of the geometry of the type II compactification (including possibly the non-Kahler resolutions of the conifold singularity) might therefore translate into detailed information about the mechanism of supersymmetry breaking in these models. The $N = 2$ models we have started with here are rather cumbersome, as they contain numerous moduli. It would be nice to find simpler examples of this phenomenon which can be more easily studied in detail. More generally, one would like to extend the class of useful $N = 1/N = 0$ dualities to models which are not obviously obtained as orbifolds of $N = 2$ dual pairs.

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