WIS/97/6, EFI-97-09, RI-1-97
hep-th/9702014

arXiv:hep-th/9702014v2 9 Feb 1997

Branes and N=1 Duality in String Theory

Shmuel Elitzur, Amit Giveon
Racah Institute of Physics
The Hebrew University
Jerusalem, 91904, Israel

David Kutasov *
Department of Physics of Elementary Particles
Weizmann Institute of Science
Rehovot, 76100, Israel

We propose a construction of dual pairs in four dimensional N=1 supersymmetric Yang-Mills theory using branes in type IIA string theory.

2/97

* On leave of absence from Department of Physics, University of Chicago, 5640 S. Ellis Ave., Chicago, IL 60637, USA.

1. Introduction.

Supersymmetric gauge theories invariant under eight or sixteen supercharges (*e.g.* N=2 and N=4 supersymmetric Yang-Mills theories in four dimensions, respectively) have been intensively studied in the last few years. Many of the diverse strong coupling phenomena that they exhibit were found to have a natural explanation in string theory (some were even discovered this way). In gauge theory, a lot is known about systems with four supercharges (in particular, N=1 supersymmetric gauge theories in four dimensions [1,2]), and it is natural to ask what string theory can teach us about such systems.

The purpose of this note is to construct, in analogy to [3], a configuration of branes in type II string theory, which appears to exhibit one of the most remarkable field theory phenomena – Seiberg's duality in four dimensional N=1 supersymmetric gauge theory [4]. This duality is the statement that two different gauge theories ("electric" and "magnetic") may sometimes give rise to the same long distance physics, *i.e.* flow to the same infrared conformal field theory. Its interpretation in string theory was recently studied in [5] from a different point of view. We will also discuss certain extensions of Seiberg's work [6].

Below, we consider a configuration of branes whose low energy worldvolume dynamics is that of an N=1 four dimensional supersymmetric Yang-Mills theory with gauge group $U(N_c)$ and N_f flavors of quarks in the fundamental representation. We focus on two microscopic coupling constants, the gauge coupling and the Fayet-Iliopoulos (FI) D-term. We show that by varying the gauge coupling (using the FI D-term to avoid a strong coupling singularity), we recover in different limits the electric and magnetic descriptions of the theory, given by Seiberg. This shows that the two models have the same moduli space of vacua and, therefore, the same chiral ring. It is natural to expect that the full infrared conformal field theories agree as well.

2. The brane configuration.

The configurations we will study involve four kinds of branes in type IIA string theory: a Neveu-Schwarz (NS) fivebrane, Dirichlet (D) sixbrane, Dirichlet fourbrane and a differently oriented NS fivebrane, which we will refer to as the NS$'$ fivebrane. Specifically, the four kinds of branes are:

(1) NS fivebrane with worldvolume $(x^0, x^1, x^2, x^3, x^4, x^5)$, which lives at a point in the (x^6, x^7, x^8, x^9) directions. The NS fivebrane preserves supercharges of the form[1] $\epsilon_L Q_L + \epsilon_R Q_R$, with

$$\begin{aligned} \epsilon_L =& \Gamma^0 \cdots \Gamma^5 \epsilon_L \\ \epsilon_R =& \Gamma^0 \cdots \Gamma^5 \epsilon_R. \end{aligned} \tag{2.1}$$

(2) D sixbrane with worldvolume $(x^0, x^1, x^2, x^3, x^7, x^8, x^9)$, which lives at a point in the (x^4, x^5, x^6) directions. The D sixbrane preserves supercharges satisfying

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^7 \Gamma^8 \Gamma^9 \epsilon_R. \tag{2.2}$$

(3) D fourbrane with worldvolume $(x^0, x^1, x^2, x^3, x^6)$ which preserves supercharges satisfying

$$\epsilon_L = \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^6 \epsilon_R. \tag{2.3}$$

(4) NS′ fivebrane with worldvolume $(x^0, x^1, x^2, x^3, x^8, x^9)$ preserving the supercharges

$$\begin{aligned} \epsilon_L =& \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^8 \Gamma^9 \epsilon_L \\ \epsilon_R =& \Gamma^0 \Gamma^1 \Gamma^2 \Gamma^3 \Gamma^8 \Gamma^9 \epsilon_R. \end{aligned} \tag{2.4}$$

It is easy to check that there are four supercharges satisfying equations (2.1)-(2.4), 1/8 of the original supersymmetry of type IIA string theory.

Similarly[2] to [3], we will study the dynamics on the worldvolume of fourbranes stretched between fivebranes. The case of interest will be a configuration of N_c D fourbranes stretched between an NS fivebrane and an NS′ fivebrane, along the x^6 direction. Thus, the worldvolume of the D fourbrane is $R^{3,1}$ times a finite interval I. The worldvolume dynamics describes at long distances an N=1 supersymmetric theory in 3+1 dimensions.

Of course, to have the possibility of stretching a D fourbrane between the NS and NS′ fivebranes without breaking supersymmetry, the two NS branes must coincide in the x^7 direction. We will see later what happens when they do not coincide.

Since the configuration of a D fourbrane between NS and NS′ fivebranes preserves four supercharges (even in the absence of D sixbranes), it describes a 3+1 dimensional

[1] Q_L, Q_R are the left and right moving supercharges of type IIA string theory in ten dimensions. They are (anti-) chiral: $\epsilon_R = -\Gamma^0 \cdots \Gamma^9 \epsilon_R$, $\epsilon_L = \Gamma^0 \cdots \Gamma^9 \epsilon_L$.

[2] T duality in the x^3 direction turns our construction into a three dimensional N=2 supersymmetric version of [3].

N=1 supersymmetric gauge theory at low energies. It is not difficult to deduce the field content. The fourbrane worldvolume fields that describe its fluctuations in the $(x^6, \cdots, x^9)$ directions are fixed by the boundary conditions at the NS fivebrane end. Those that describe fluctuations in the $(x^4, \cdots, x^7)$ directions are fixed by boundary conditions at the NS$'$ fivebrane end. Thus, the only massless mode arising from 4-4 strings is the $U(N_c)$ worldvolume gauge field.

To add matter, we insert (as in [3]) N_f D sixbranes at values of x^6 that are between the positions of the NS and NS$'$ fivebranes. The 4-6 strings describe N_f chiral multiplets in the fundamental representation of $U(N_c)$. Note that:

1) The relative position of the NS and NS$'$ fivebranes in the x^7 direction plays the role of a FI D-term in the $U(N_c)$ gauge theory on the fourbranes. To find a supersymmetric vacuum when it is non-vanishing, one has to turn on Higgs expectation values for the quarks. This can be achieved by a similar mechanism to that described in [3], whereby a D fourbrane stretched between the NS and NS$'$ fivebranes touches a D sixbrane (at which point the mass of the corresponding chiral multiplet vanishes), and splits into two fourbranes, one stretched between the NS and D sixbranes, and the other between the D sixbrane and the NS$'$ fivebrane.
2) The distance between the NS and NS$'$ fivebranes in the x^6 direction, L_6, determines the 3+1 dimensional fourbrane worldvolume gauge coupling: $1/g_4^2 \propto L_6$.
3) The distance in the (x^4, x^5) directions between the NS$'$ fivebrane and the N_f D sixbranes determines the masses of the N_f chiral multiplets.
4) It is possible for a fourbrane stretched between the NS and NS$'$ fivebranes to break on a D sixbrane into two pieces with a relative splitting in the (x^8, x^9) directions. This corresponds to turning on a Higgs expectation value for one of the quarks.

To summarize, the brane configuration we start with, viewed along the x^6 direction, is the following: the NS fivebrane is the leftmost object, and is connected to the NS$'$ fivebrane by N_c fourbranes. The D sixbranes are placed between the NS and NS$'$ branes. This configuration corresponds to the electric theory of [4]. In the next section we will describe the magnetic one.

3. Seiberg's duality.

The dynamics on the fourbrane worldvolume describes an N=1 supersymmetric $G_e = U(N_c)$ gauge theory coupled to N_f fundamental "quarks" in 3+1 dimensions. Seiberg's

duality [4] is the statement that at long distances this system has another description, where the gauge group is replaced by $G_m = U(N_f - N_c)$, and in addition to the N_f chiral superfields in the $\mathbf{N_f} - \mathbf{N_c}$ of $U(N_f - N_c)$ there appears a G_m singlet field M, in the $\mathbf{N_f^2}$ of the $U(N_f)$ global symmetry group (here this is the D sixbrane gauge group, which gives a global symmetry on the fourbrane worldvolume). The "magnetic meson field" M couples to the magnetic quarks via a superpotential which will be described later.

To find this "magnetic" description, we follow [3] and move the NS fivebrane to the other side of the NS$'$ fivebrane in the x^6 direction. This motion of the NS fivebrane in the x^6 direction corresponds to changing the microscopic gauge coupling of the theory, and thus should not change the infrared behavior. There is a potential singularity corresponding to the point where the NS and NS$'$ fivebranes coincide (and the coupling in the gauge theory diverges), but that can be avoided (for $N_f \geq N_c$) by switching on and off the FI D-term as we vary the gauge coupling or, equivalently, going around the NS$'$ fivebrane in the (x^6, x^7) plane. As we shall see, after the NS fivebrane completes its motion, the system one finds is the magnetic description of the theory.

To facilitate the presentation, it is convenient to think of the motion described above as composed of the following four steps:

(a) The NS fivebrane moves to the right in the x^6 direction, crossing all N_f D sixbranes.

(b) The NS fivebrane moves away from the NS$'$ fivebrane in the x^7 direction.

(c) The NS fivebrane moves in the x^6 direction, to the other side of the NS$'$ fivebrane.

(d) The NS fivebrane moves in the x^7 direction back to its original x^7 position.

We will next discuss in turn the steps (a) – (d), focusing on the different phenomena that take place.

Step (a): Whenever the x^6 location of the NS fivebrane passes through that of a D sixbrane, the two branes actually meet in space[3]. This leads [3] to the creation of a new fourbrane connecting the NS fivebrane (now on the right of the D sixbrane) and the D sixbrane. Thus at the end of step (a), in addition to the N_c fourbranes connecting the NS and NS$'$ fivebranes, there are N_f fourbranes connecting the NS fivebrane to the N_f D sixbranes (all of which are now to the left of the NS fivebrane).

Step (b): As mentioned above, the N_c fourbranes stretched between the NS and NS$'$ fivebranes preserve supersymmetry only when the x^7 positions of the two branes are the same. Since we now want to move the NS fivebrane relative to the NS$'$ fivebrane in the x^7

[3] In contrast, the NS$'$ fivebrane and D sixbrane can avoid each other in the (x^4, x^5) directions.

direction, in order not to break SUSY the gauge theory must be in a Higgs phase, which in the brane language means the following. If the D sixbranes are at the same (x^4, x^5) as the NS$'$ fivebrane (which means that the electric quarks Q are massless), then the N_c fourbranes connecting the NS and NS$'$ fivebranes can connect to N_c of the N_f new fourbranes created in step (a) and together leave the NS fivebrane in the inverse of the process described in [3], Figure 3a. Of course, for this to be possible, one must have $N_f \geq N_c$, a constraint related both to the fact [7] that SQCD has no supersymmetric vacuum for $N_f < N_c$, and to similar observations in [3] about the theory with eight supercharges. At the end of step (b) we thus find the following situation. There are N_c fourbranes connecting the NS$'$ fivebrane to N_c D sixbranes. The other $N_f - N_c$ D sixbranes are connected by fourbranes to the NS fivebrane.

Step (c): Since the NS and NS$'$ fivebranes do not meet in space (even when their x^6 values coincide), nothing special is expected to happen at this stage.

Step (d): As the NS fivebrane comes back to its original x^7 position, the $N_f - N_c$ fourbranes connecting it to $N_f - N_c$ different D sixbranes touch the NS$'$ fivebrane. They then split into $N_f - N_c$ fourbranes stretched between the NS and NS$'$ fivebranes, and $N_f - N_c$ fourbranes stretched between the NS$'$ fivebrane and $N_f - N_c$ D sixbranes.

The final brane configuration is the following. The NS fivebrane is connected to the NS$'$ fivebrane by $N_f - N_c$ fourbranes. The NS$'$ fivebrane is further connected by N_f fourbranes to the N_f D sixbranes. To see that this is the magnetic description of the original theory, note that:

1) The gauge group on the fourbrane worldvolume is $G_m = U(N_f - N_c)$.
2) Unlike the case of [3], the N_f fourbranes connecting the D sixbranes to the NS$'$ fivebrane are not rigid – they can fluctuate in the (x^8, x^9) directions. There are $N_f \times N_f$ complex fields $M_i^{\tilde{i}}$ $(i, \tilde{i} = 1, \cdots, N_f)$ coming from 4-4 strings, parametrizing these fluctuations. These are Seiberg's magnetic mesons.
3) The coupling of the magnetic mesons M to the magnetic quarks q is as expected [4], through a magnetic superpotential:

$$W_m = M_i^{\tilde{i}} q^i \tilde{q}_{\tilde{i}}. \tag{3.1}$$

This is clear from the geometry of open strings stretched between various branes.

4) One can check that deformations of the electric theory agree with those of the magnetic one, in precisely the way described in [4]. In particular, turning on a quark mass in

the electric description, which corresponds to moving the D sixbranes relative to the NS′ fivebrane in the (x^4, x^5) directions, corresponds, after following the path (a)-(d) above, to turning on Higgs expectation values in the magnetic description. Turning on an expectation value for the electric quarks corresponds in the magnetic description to giving the "magnetic mesons" M an expectation value.

4. Duality with an adjoint superfield.

Seiberg's original work was generalized in [6] to theories with a single adjoint field. In [6] it was shown that the "electric" theory with gauge group $G_e = SU(N_c)$, N_f flavors of fundamental matter $Q_i, \tilde{Q}^{\tilde{i}}$, and a single adjoint field X with superpotential

$$W_e = \mathrm{Tr} X^{k+1}, \tag{4.1}$$

is equivalent at long distances to a magnetic theory with gauge group $G_m = SU(kN_f - N_c)$, with an adjoint field Y, N_f flavors of magnetic quarks q^i, $\tilde{q}_{\tilde{i}}$, and k magnetic mesons M_j, each of which has N_f^2 components. The full magnetic superpotential is (roughly – see [6] for more details):

$$W_m = \mathrm{Tr} Y^{k+1} + \sum_{j=1}^{k} M_j \tilde{q} Y^{k-j} q. \tag{4.2}$$

It is not difficult to guess the right brane configuration describing this duality. If we replace the single NS fivebrane of section 2 by k coincident NS fivebranes[4] all connected to a single NS′ fivebrane, and repeat the analysis of section 3, we find that the gauge group is transformed appropriately[5], $G_e = U(N_c) \to G_m = U(kN_f - N_c)$, and the right set of magnetic mesons M_j (4.2) appears. We will not go through the analysis of section 3 for this case; in deriving the result it is important to note that configurations where more than one fourbrane connects the NS′ fivebrane to a given D sixbrane are supersymmetric. In contrast, in [3] it has been pointed out that to reproduce the expected field theory behavior, configurations with more than one fourbrane connecting a given NS fivebrane to a given D sixbrane (s-configurations) should not preserve supersymmetry. Geometrically, the difference might be due to the fact that two D fourbranes connecting an NS fivebrane

[4] Note that configurations of coincident NS fivebranes appear in type II string theory near A_k singularities in the moduli space of K3 compactifications [8-10].

[5] The duality of [6] can be extended from $SU(N)$ to $U(N)$ gauge groups.

to a D sixbrane must necessarily be on top of each other, while different D fourbranes connecting an NS$'$ fivebrane to a D sixbrane may be separated in the (x^8, x^9) directions, which are common to the two branes.

As discussed in detail in [6], one way to understand the generalized duality is to resolve the superpotential (4.1) to:

$$W_e = \mathrm{Tr} \sum_{j=2}^{k+1} a_j X^j; \quad a_{k+1} = 1. \tag{4.3}$$

For generic values of the $\{a_j\}$, the bosonic potential $V \sim |W'|^2$ has k distinct minima, in each of which the adjoint field is massive. If r_j of the N_c eigenvalues of X sit in the j'th minimum of V ($\sum r_j = N_c$), the gauge symmetry is spontaneously broken:

$$U(N_c) \to U(r_1) \times U(r_2) \times \cdots \times U(r_k). \tag{4.4}$$

The only massless matter near the j'th minimum is N_f fundamentals of $U(r_j)$; thus for generic $\{a_j\}$, the model describes at low energies k decoupled supersymmetric QCD theories with gauge groups $U(r_j)$, (4.4). One can apply Seiberg's duality to each of the $U(r_j)$ factors, taking them to $U(N_f - r_j)$, as in section 3. Finally, in the limit $a_j \to 0$, the full magnetic gauge group is restored: $U(N_f - r_1) \times \cdots \times U(N_f - r_k) \to U\left(\sum_j (N_f - r_j)\right) = U(kN_f - N_c)$.

The same story can be told using the brane configuration described above. One can group the N_c fourbranes stretched between the k NS fivebranes and the NS$'$ fivebrane into groups of r_j fourbranes stretched between the j'th NS fivebrane and the NS$'$ fivebrane. The superpotential (4.3) can be parametrized by the locations of k points in a plane (the k complex solutions of $W'(x) = 0$). On the other hand, the position of each NS fivebrane is specified by four real numbers. Translation of the k NS fivebranes in the (x^6, x^7) directions correspond to explicit breaking of $U(N_c)$ as in (4.4), obtained by assigning different gauge couplings and/or FI D-terms to the different $U(r_j)$. The superpotential is encoded in the position of the k NS fivebranes in the (x^8, x^9) plane. In fact, one can think of the adjoint field X (4.1) as the 4-4 string describing (infinitesimal) fluctuations of the fourbrane in the $(8, 9)$ directions. Note that in the case where the $U(N_c)$ gauge symmetry is broken as in (4.4), with k decoupled SQCD systems describing the different minima of the superpotential (4.3), there are parameters in the gauge theory that can not be seen in the string (brane) description, corresponding to changing the masses of the quarks around

each minimum independently (in the brane configuration, all the masses are set to be equal).

5. Comments.

1) It is clear from the brane construction of sections 2, 3 that the electric and magnetic theories have the same moduli space of vacua. This is equivalent to the statement that the two models have the same ring of chiral operators. Since the models in question do not have a Coulomb phase, it is likely that the relation extends to the full infrared conformal field theory.
2) There was some recent work on N=1 duality in string theory, from the point of view of F-theory [5]. In the case of systems with eight supercharges, one can show [11] that the analog of our approach is related by T-duality to the study of threebranes in F-theory. Specifically, consider type IIB string theory compactified on a $K3$ surface (it is convenient to consider $K3 \simeq (S^1)^4/Z_2$, as a starting point), in the presence of sevenbranes wrapping the $K3$ and threebranes at points on $K3$. T-duality on a single S^1 turns [9] this into a type IIA string compactified on a T^4/Z_2 orbifold, with sixteen NS fivebranes located (for generic values of the moduli of $K3$) at generic points on T^4/Z_2. The threebranes and sevenbranes are transformed under T-duality into fourbranes and sixbranes, respectively. For the case of theories with four supercharges, similar considerations of type IIB on $T^6/Z_2 \times Z_2$ lead to our brane configuration.
3) T duality in the x^3 direction turns the brane configuration presented in section 2 into a three dimensional N=2 supersymmetric version of the construction of [3]. It would be interesting to use the brane picture to study the dynamics of the gauge theory on the threebrane, and in particular explore the consequences of the $SL(2,Z)$ duality group of type IIB string theory. This may lead to generalizations of mirror symmetry [12,13,14,3] to N=2 gauge theory in 2+1 dimensions.

Acknowledgements: We thank A. Schwimmer for useful discussions. This work is supported in part by the Israel Academy of Sciences and Humanities – Centers of Excellence Program. The work of A. G. is supported in part by BSF – American-Israel Bi-National Science Foundation. S. E. and A. G. thank the Einstein Center at the Weizmann Institute for partial support.

References

[1] N. Seiberg, hep-th/9408013; hep-th/9506077.

[2] K. Intriligator and N. Seiberg, hep-th/9509066.

[3] A. Hanany and E. Witten, hep-th/9611230.

[4] N. Seiberg, hep-th/9411149, Nucl. Phys. **B435** (1995) 129.

[5] M. Bershadsky, A. Johansen, T. Pantev, V. Sadov and C. Vafa, hep-th/9612052; C. Vafa and B. Zwiebach, hep-th/9701015.

[6] D. Kutasov, hep-th/9503086, Phys. Lett. **351B** (1995) 230; D. Kutasov and A. Schwimmer, hep-th/9505004, Phys. Lett. **354B** (1995) 315; D. Kutasov, A. Schwimmer and N. Seiberg, hep-th/9510222, Nucl. Phys. **B459** (1996) 455.

[7] I. Affleck, M. Dine and N. Seiberg, Nucl. Phys. **B241** (1984) 493.

[8] A. Strominger, hep-th/9512059, Phys. Lett. **383B** (1996) 44.

[9] D. Kutasov, hep-th/9512145, Phys. Lett. **383B** (1996) 48.

[10] E. Witten, hep-th/9512219, Nucl. Phys. **B463** (1996) 383.

[11] S. Elitzur, A. Giveon and D. Kutasov, unpublished.

[12] K. Intriligator and N. Seiberg, hep-th/9607207, Phys. Lett. **387B** (1996) 513.

[13] J. de Boer, K. Hori, H. Ooguri and Y. Oz, hep-th/9611063; J. de Boer, K. Hori, H. Ooguri, Y. Oz and Z. Yin, hep-th/9612131.

[14] M. Porrati and A. Zaffaroni, hep-th/9611201.