Light propagation control by finite-size effects in photonic crystals

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Abstract

We exhibit the strong influence on light propagation of the finite size in photonic band-gap material. We show that light emission can be controlled by the symmetry group of the boundary of the finite device. These results lead simply to important practical applications.

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Photonic crystals are expected to permit the realization of devices for integrated optics or laser microcavities [1–5]. The today technology now authorizes the making of bidimensional photonic crystals in the optical range [6]. From the theoretical point of view, there are two ways for characterizing PBG materials. The first way is the one commonly used in solid state physics where pseudo-periodic boundary conditions are imposed leading to Bloch waves theory. This is a powerful tool for the computation of the band structure of the material [7]. However, this theory cannot be used when dealing with the scattering of an electromagnetic wave by a finite device (which is the actual experimental situation) [8–10]. In that case, boundary effects cannot be skipped and must be taken into account. In this letter, we address the problem of the effect on light propagation of both the symmetry group of the lattice and the symmetry group of the boundary of a finite piece of the material. The general study of the symmetry of the lattice modes has already been addressed in order to understand the possible degeneracies and the existence of full band gaps [11–15]. Here, we show that the finite size of the device strongly modifies the behaviour of the electromagnetic field, and that the symmetry of the boundary is a crucial parameter for the control of light emission from PBG devices.

We deal with a bidimensional photonic crystal that is made of a finite collection of parallel dielectric rods of circular section. The rods are settled according to a lattice with some symmetry group $\mathbf{G}_Y$ (Y denotes the elementary cell) of the plane. The relative permittivity of the lattice is a Y-periodic function $\varepsilon_r(x,y)$. The rods are contained in a domain Ω, having a symmetry group $\mathbf{G}_\Omega$. As we deal with objects that are embebbed in an affine euclidean space, both groups must be given as subgoups of the group of plane isometries $\mathbb{O}(2)$, in a canonical oriented basis. This is due to the fact that the use of the abstract groups does not permit to distinguish between two isomorphic realizations. Indeed, using the abstract groups and unspecified representations of degree 2, we could not, for instance, distinguish between two squares $C_{1,2}$ deduced from one another through a rotation r of angle $\pi/4$: denoting Γ_1 a realization of D_4 as an invariance group of C_1, then $\Gamma_2 = r\Gamma_1 r^{-1}$ is a representation of C_2

and therefore the equivalence class of Γ_1 is not sufficient to compare the two figures and a canonical basis has to be precised.

Denoting 1_Ω the characteristic function of Ω (which is equal to 1 inside Ω and 0 elsewhere), the relative permittivity is given by $\varepsilon_\Omega(x,y) = 1 + 1_\Omega(\varepsilon_r(x,y) - 1)$. Assuming a s-polarized incident field (electric field parallel to the rods), the total electric field verifies the d'Alembert equation:

$$c^{-2}\partial_{tt}E_z = \varepsilon_\Omega^{-1}\Delta E_z \tag{1}$$

Our aim is to study the invariance of this equation under the action of the various groups characterizing the geometry of the problem. For arbitrary cross-sections of the fibers, we should also introduce their symmetry group (for instance the group D_4 for square rods). The choice of circular rods simplifies the study in that their symmetry group $\mathbb{O}(2)$ contains both $\mathbf{G}_Y$ and $\mathbf{G}_\Omega$. Note however that strong effects can be obtained by using peculiar symmetries of the rods, including the enlargment of the band gaps [16]. Let us now denote $\Gamma(\mathbf{G}_Y)$ and $\Gamma(\mathbf{G}_\Omega)$ the groups of operators associated to $\mathbf{G}_Y$ and $\mathbf{G}_\Omega$ respectively [17]. Both operators Δ and ∂_{tt} commute with $\Gamma(\mathbf{G}_Y)$ and $\Gamma(\mathbf{G}_\Omega)$. However, due to the function ε_Ω, the propagation equation is only invariant under the intersection group $\Gamma(\mathbf{G}_Y) \cap \Gamma(\mathbf{G}_\Omega)$. This simple remark is a crucial point in understanding the invariant properties of finite crystals and it leads to extremely important effects in practical applications: it is a clue to controlling the directions of light propagation in the structure. Indeed, due the boundary of Ω, the degree of the global symmetry group of the device is reduced and consequently, from selection rules, the number of forbidden directions increases.

In order to make this reasoning more explicit, we present two numerical experiments obtained through a rigorous modal theory of diffraction [10,18] that takes into account all of the multiple scattering between rods (in case of an infinite lattice this is exactly the KKR method), moreover it has been succesfully compared with experiments [19]. This method allows to deal with finite structure without periodizing tricks [20] that may lead to spurious phenomena [21]. As a lattice group $\mathbf{G}_Y$, we choose the diedral group D_6 [22], so that the lattice has a hexagonal symmetry. The distance between neighbouring rods is denoted by d. In order to create a defect mode in that structure, we open a microcavity at the centre of the crystal by removing a rod (see [8,9,12,23,24] for studies of the properties of defects in photonic crystals). A defect mode appears within the first gap at a complex wavelength $\lambda/d = 2.264 + 0.06i$ (using a harmonic time-dependence of $e^{-i\omega t}$). Such a structure can be used as a resonator coupled to waveguides [14].

In the first experiment, we choose the same group for the boundary as that of the lattice (fig.1)(i.e. $\mathbf{G}_\Omega = D_6$). In that case the propagation equation is completely invariant under $\Gamma(D_6)$. We plot the map of the Poynting vector modulus associated to the defect mode (fig.1) and the radiation pattern is given in fig.2. Clearly the field shows a hexagonal symmetry, which is obvious from the invariance of the d'Alembert equation. However, when designing light emitting devices, one whishes to control the direction of light emission. In this example, there are too many directions of emissions: such a device is to be coupled, for instance to waveguides, and to get a good transmission ratio, one needs to concentrate the field in a few useful directions. As it has been stated above, the number of authorized directions can be reduced by reducing the global symmetry group D_6 of the device. This is what we do in the next numerical experiment where we have changed the boundary so that it has now a rectangular symmetry ($\mathbf{G}_\Omega = \{e, s_x, s_y, r\}$, where s denotes a symmetry with respect to x and y respectively and r is a rotation of angle π), the device is depicted in fig. 3. In that particular case, the group of the boundary is contained in the group D_6. Then the equation (1) is no longer invariant under $\Gamma(D_6)$ but solely under $\Gamma(D_6) \cap \Gamma(\mathbf{G}_\Omega) = \Gamma(\mathbf{G}_\Omega)$ which is strictly contained in $\Gamma(D_6)$. All the other transformations are now forbidden. That way, we expect a strong reduction of the directions of propagation of the field.

Indeed, the map of the Poynting vector of the defect mode (fig. 3) as well as the radiation pattern (fig. 4) shows a strong enhancement of the vertical direction by forbidding the transverse directions linked to the rotations and the oblique symmetries. We have designed a resonator that permits to couple the radiated field in up and down directions with a better efficiency.

It should be noted that a group theoretic analysis gives only informations on the possible directions of emission, the actual directions on which the field concentrates cannot be obtained by this mean: a rigorous computation involving a finite structure is then needed. Nevertheless, we have demonstrated that it was possible to strongly increase the efficiency of resonators by simply taking into account the symmetry of the boundary of the device. This remark can be used rather easily in experimental situations and could lead to a dramatic enhancement of the output of PBG based devices.

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Figure captions

Figure 1: Map of the Poynting vector modulus of the defect mode. Both the lattice and the boundary have the same symmetry group ($\mathbf{G}_\Omega = D_6$). The red line represents the hexagonal symmetry of the boundary of the crystal. The defect mode possesses all transformations of the hexagonal point group. The ratio of the rod radius to the spatial periode is $r/d = 0.15$ and the optical index is $n = 2.9$.

Figure 2: Radiation pattern of the defect mode for the crystal defined in figure 1. The radiated power is invariant by the hexagonal point group.

Figure 3: Map of the Poynting vector modulus of the defect mode. The global symmetry of the crystal is given by the subgroup $\mathbf{G}_\Omega = \{e, s_x, s_y, r\}$. The red line represents the rectangular symmetry of the boundary of the crystal. The defect mode is invariant under $\mathbf{G}_\Omega$. The ratio of the fiber radius to the spatial periode is $r/d = 0.15$ and the optical index is $n = 2.9$.

Figure 4: Radiation pattern of the defect mode for the crystal defined in figure 3. The radiated power is invariant by the subgroup $\mathbf{G}_\Omega$.

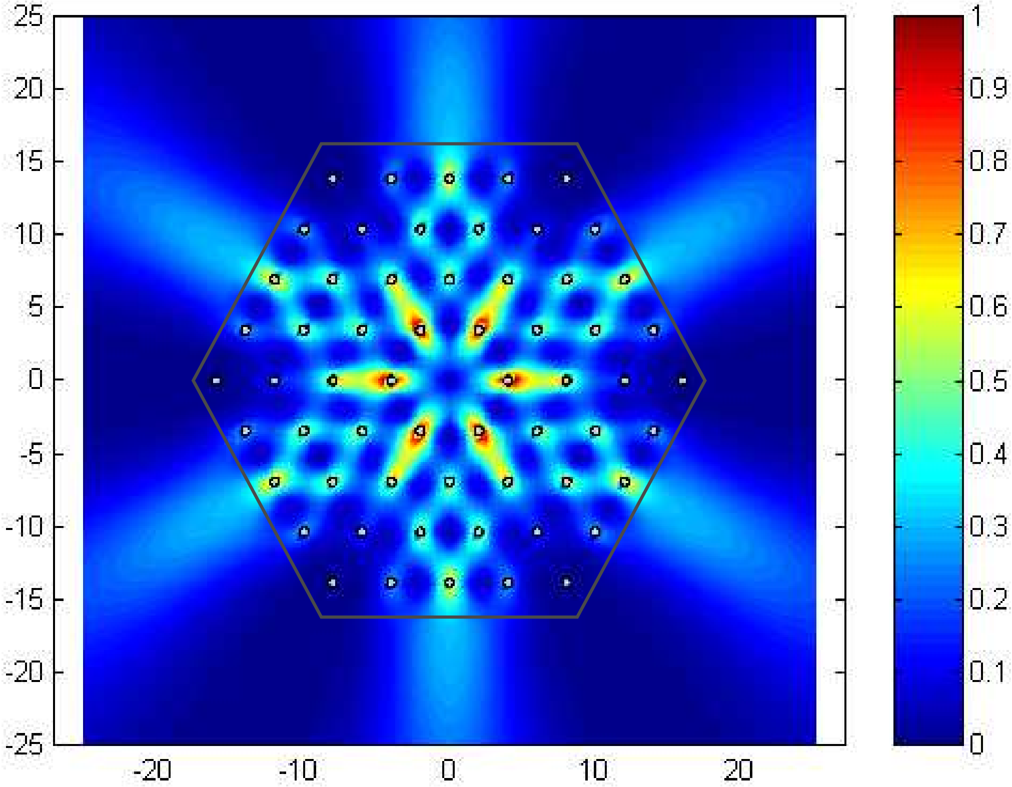

Figure 1

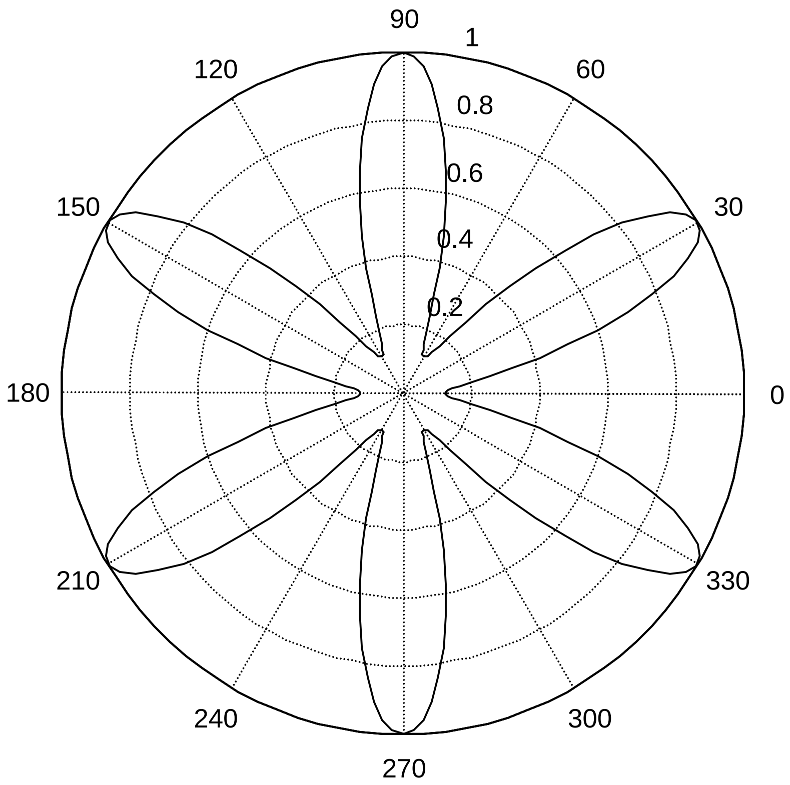

Figure 2

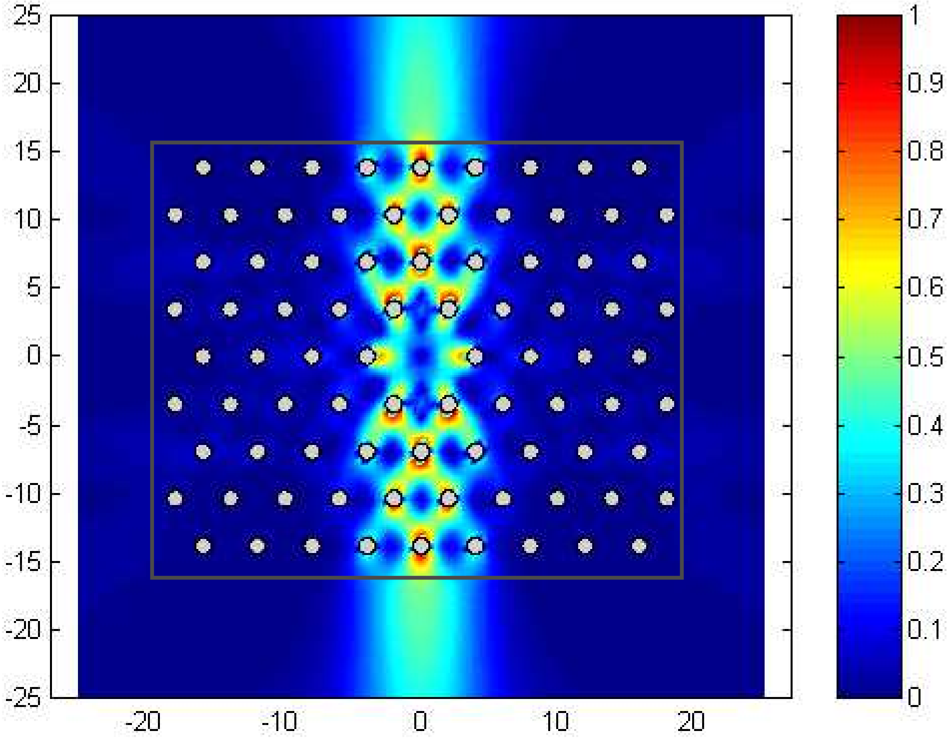

Figure 3

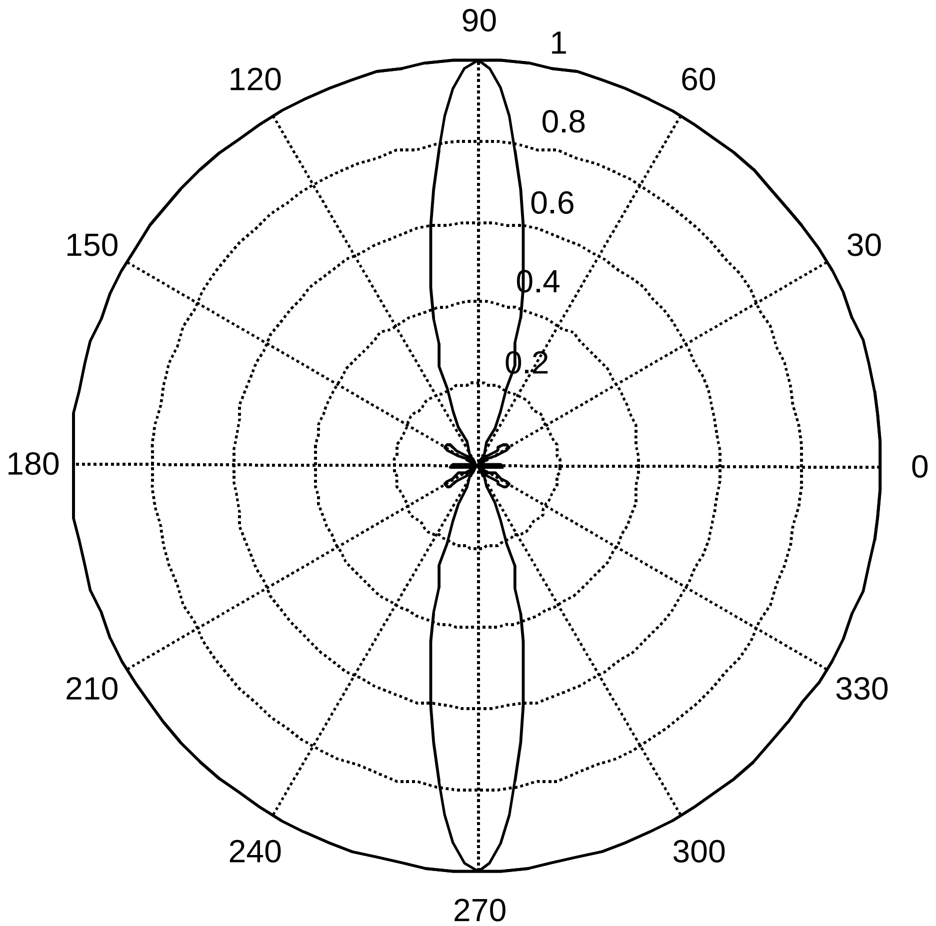

Figure 4