Scattering of the acoustic waves on finite cylindrical covers

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November 17, 2018

The interaction problems of wave fields with resilient bounded bodies arise in the different fields of mechanics, hydro acoustics, geophysics and seismology.

The analytical solutions of the problems of hydro acoustic waves scattering by resilient bodies are directly connected with the methods development of modeling wave fields interaction processes with underwater objects. Increasing the complexity of underwater objects geometry we get the increase of mathematical complexity of the pressure field determination problem solution. In this connection the problems are solved with simplified mathematical statement, which analysis helps to indicate the significant features of geometry and structure of modeling object. Up till now number of analytical and experimental researches were implemented according to the statement of the problem. These researches were directed to the conformities determination of influence of any construction or technologic property on structure parameters of reflected signals.

The problems of scattering of acoustic pressure waves by resilient bodies with complex geometry have no analytical solution.

Applying the direct numerical methods (e.g. finite difference method) to solve partial differential equations of these problems is not expedient even using powerful computers due to multi-dimensional type of the problem. For this reason high-frequency and low-frequency methods are often used to solve such problems. But when wavelength is comparable by order with the characteristic size of disperse body the frequency lies in moderate range. Exactly this range as shown in [16] is highly informational.

To solve the scattering problems in the moderate frequency range the limited integral equations method is applied.

Applying Grin theorem let us reduce the solution of stationary problem of resilient body dispersion to the solution of combined singular equations for the system of potential displacements. The solution of the above determines the surface potentials, which are used to define the outside field. Thus, the number of dimension variables is reduced and it becomes possible to use numerical methods.

In works [17, 1] the problem of sound diffraction on the finite cylinder with mixed boundary conditions is solved by Grin's function method. In [18] the axis-symmetrical dispersed pressure of resilient disk is computed using finite element method and HelmHoltz integral equation.

As is well known, there is a tight relation between the solution of scattering and emission problems. It could be defined by reciprocity theorem [19]. This relation could be used to transfer emission problem results to scattering problems and vise versa [20].

While solving complex problems, for example three-dimensional problems of acoustic waves scattering on the resilient bodies or two-dimensional problems of scattering on the piece-wise-smooth resilient bodies and problems of scattering of acoustic waves on the thin-wall covers, troubles with definite satisfaction of the appropriate boundary conditions arise.

The considered work is devoted to the research of scattered and emissed acoustic waves by finite cylindrical covers, resilient cylinders and round plates.

The presence of ribs fracture line in bottoms interface and cylindrical covers on the face of resilient cylinder and on edge of round plates inputs the additional troubles of evaluating of such problems.

During last years for the solution of space problems of scattering of acoustic waves numerical and integral equations methods are effectively applied. Despite the achieved researches of applying these methods to three-dimensional problems, questions of efficiency increase of existing methods and their PC-realization are still actual. In particular, one of the hydro-acoustic problems is the evaluation of scattered and emitted pressure field by three-dimensional bodies of cylindrical form. But during the scattered pressure field exploration near bounded disperser with edges and also internal features of dissipater troubles arise, which are hardly solved without analytical research. That is why the important problem of analytic-numerical methods development and development of software for acoustic space interaction with finite bodies of cylindrical form are actual.

During the solution of problems of emission and scattering of acoustic waves by bounded bodies of cylindrical form we use the strategy based on applying of bounded integral equations method combined with series method and later using the improvement of convergence method, which consider particularities of desired functions. The proposed strategy is authorized by the fact that the series method is effective to solution of equations of resiliency theory, thin covers and plates, and the bounded integral equations method is effective to research of infinite acoustic environment vibrations. Using this methodology let us describe the behavior of bounded resilient bodies near salient line, explore the pressure field in near and far zone, obtain the solution of the problem with defined precision in wide frequency range.

In the first chapter basic linear equations of motion and initial relations are considered, statement and solution of problem of scattering of flat acoustic wave by finite cylindrical cover bounded at the ends by resilient bottoms is done.

The improvement of convergence of obtained infinite systems of linear algebraic equations was held, using asymptotic properties of Fourier decomposition

coefficients of the desired functions while solving the equations of cover theory.

The solutions of the problem of scattering of flat pressure wave by open-end free cylindrical cover, and also by tension cylindrical cover with defined inner tensions. In case of thin resilient cylindrical cover and round plates its motion is modeled by the linear Thimoshenko model.

Today the problem of learning of dynamical processes of acoustic fields interaction with thin-walled structures in liquids is of great interest. The theoretical exploration of these processes is strongly associated with building of effective analytic-numerical methods of mathematical physics equation solving, which are realized on today's PC. For example, while analyzing wave diffraction on solid bodies and covers of revolution with fixed ends in [21] finite-element and spline-functions methods based on function approximation theory were applied.

We assume that scattering objects could easily move in boundless liquid environment or to be deformless.

In this work we propose the analytical method of dynamical problems solution of fixed oscillation of infinite acoustic environment interacted with resilient finite cylindrical bodies.

In the nowadays technics problems of oscillation of finite cylindrical covers, filled with liquid, their sound waves emission at vibration in infinite acoustic environment are actual.

While interaction of wave fields with resilient bounded bodies problems arise in different fields of technics, medicine, hydro-acoustics and geo-physics.

The analytical solutions of problems of scattering of acoustic waves by resilient bodies are directly related with developing methods of modelling of interaction processes of wave fields with underwater objects.

Let in the infinite ideal compressible rigid media the cylindrical cover of finite length bounded at the ends by bottoms is placed. Cover is filled with ideal compressible fluid inside with density ρ_0 and sound velocity c_0.

The considered cover is referred to cylindrical system of coordinates r', z', Θ where axis oz' is aligned with axis of cylindrical cover and plane $z' = 0$ is equidistant to ends (see Fig. 1). Flat harmonic in time pressure wave falls to the cover (see Fig. 1):

$$P^{\alpha}(r, z, \Theta, \tau) = P_0 \exp\left(-i\omega\left(r\cos\Theta\sin\varphi^* + z\cos\varphi^* - \tau\right)\right) \qquad (1)$$

In formula (1) and below following symbolism is used: $r = \frac{r'}{R_0}; z = \frac{z'}{R_0}$ are nondimentional coordinates, $\tau = \frac{ct}{R_0}; \omega = \frac{\Omega R_0}{c}$ are nondimentional time and angular frequency, c is sound velocity in the dissipating body environment, R_0 is radius of middle surface of cylindrical cover, t, Ω are time and angular frequency, φ^* is angle between wave direction and axis of the cylindrical cover, P_0 is constant with the dimension of pressure. Time factor $\exp(i\omega\tau)$ is omitted below.

The problem of definition of dissipated pressure field outside the cover $P^e(r, z, \Theta)$ and in filler $P^0(r, z, \Theta)$ and components of cover movement vector $\vec{U}$ and plates $\vec{W}$ is reduced to solution of system of differential equations.

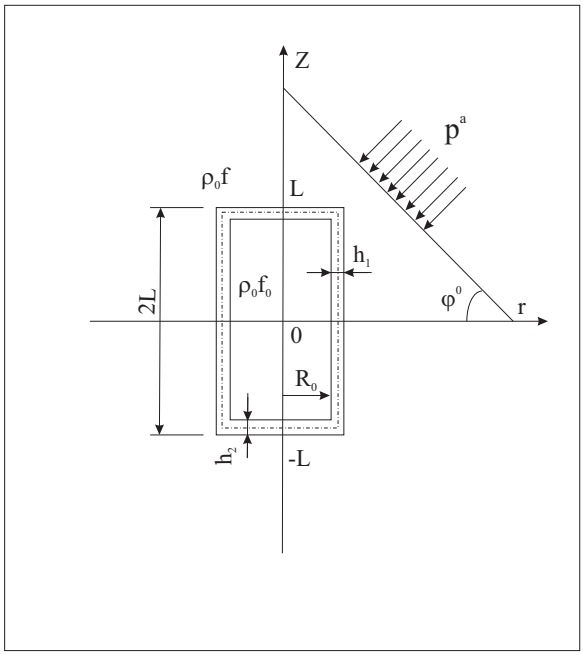

Fig. 1

Dissipated acoustic pressure $P^e\left(r,z,\Theta\right)$ in unbounded acoustic medium is described by wave equation [1]

$$\left(\Delta+\varpi^2\right)P^e\left(r,z,\theta\right)=0 \tag{2}$$

where Δ is three-dimensional Laplace operator.

Dissipated pressure in fluid which fills the cover $P^0\left(r,z,\Theta\right)$ satisfies the wave equation [2]

$$\left(\Delta+\frac{\varpi^2}{\beta_0^2}\right)P^o\left(r,z,\theta\right)=0 \tag{3}$$

where $\beta_0^2=\frac{c^2}{c_0^2}$.

The dynamics of thin resilient cylindrical cover is described by linear equation of Timoshenko cover theory which takes into account rotary inertia and deformation of transversed shift [2]

$$L_{ij}u_j=g_1\delta_{i3};\left(L_{ij}=L_{ji};i,j=1,2,3,4,5\right) \tag{4}$$

where δ_{ij} is the Cronicler's symbol,

$$\begin{aligned}
L_{11} &= \left(1+a^2\right)\left(\frac{\partial^2}{\partial\theta^2}-\varkappa_1^2\right)+\sigma_1\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2};\\
L_{12} &= a^2\left(-\frac{\partial^2}{\partial\theta^2}+\varkappa_1^2+\sigma_1\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2}\right)+\varkappa_1^2
\end{aligned}$$

$$
\begin{aligned}
L_{22} &= a^2\left(\frac{\partial^2}{\partial\theta^2}-\varkappa_1^2+\sigma_1\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2}\right)-\varkappa_1^2;\\
L_{13} &= \left(1+a^2\right)\left(1+\varkappa_1^2\right)\frac{\partial}{\partial\theta};L_{14}=\sigma_2\frac{\partial^2}{\partial z\partial\theta};L_{15}=L_{24}=0;
\end{aligned}
$$

$$
\begin{aligned}
L_{23} &= \left[\left(1+a^2\right)\varkappa_1^2+a^2\right]\frac{\partial}{\partial\theta};L_{25}=a^2\sigma_2\frac{\partial^2}{\partial z\partial\theta};\\
L_{33} &= \left(1+a^2\right)\left(-\varkappa_1^2\frac{\partial^2}{\partial\theta^2}+1\right)-\varkappa_1^2\frac{\partial^2}{\partial z^2}-\frac{\varpi^2}{\beta^2};L_{34}=\nu_{01}\frac{\partial}{\partial z};
\end{aligned}
$$

$$
\begin{aligned}
L_{35} &= -\varkappa_1^2\frac{\partial}{\partial z};L_{44}=\left(1+a^2\right)\sigma_1\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2};\\
L_{45} &= a^2\left(-\sigma_1\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2}\right);\sigma_1=\frac{1-\nu_{01}}{2};\sigma_2=\frac{1+\nu_{01}}{2};
\end{aligned}
$$

$$
\begin{aligned}
L_{55} &= a^2\left(\sigma_1\frac{\partial^2}{\partial\theta^2}+\frac{\partial^2}{\partial z^2}+\frac{\varpi^2}{\beta^2}\right);\beta^2=\frac{c_{10}^2}{c^2};\varkappa_1=\frac{c_{20}}{c_{10}};\\
g_1 &= \frac{g_r}{\xi\rho c^2};\xi=\frac{\beta^2h_1\rho_1}{R_0\rho};
\end{aligned}
$$

$$
a^2=\frac{h_1^2}{R_0^2};c_{10}^2=\frac{E_1}{\rho\left(1-\nu_{01}^2\right)};c_{20}^2=\frac{E_1k_{T1}}{2\rho_1\left(1+\nu_{01}\right)}.
$$

E_1,ν_{01},ρ_1,h_1 is Young's modulus, Poisson's ratio, density and thickness of cover material, ρ is density of outside liquid medium, U_2,U_4 are turning angle of normal of cover's middle surface in planes $r\Theta$ and rz appropriately; g_r is radial component of outside force falls on middle surface area unit.

Differential equations of Timoshenko-Mindlin which describe the transverse motion of round resilient plates with shifted and rotary inertia taken into account and characterizing few first asymmetrical thickness mod oscillations have the following form [3]

$$
\left(\Delta_0^2+\alpha_1\Delta_0-\alpha_2\right)W_z\left(r,\theta\right)=\left(\alpha_2^*-\alpha_1^*\Delta_0\right)g_z \tag{5}
$$

$$
\left(\Delta_0+k_3^2\right)\Phi\left(r,\theta\right)=0 \tag{6}
$$

where

$$
\begin{aligned}
\Delta_0 &= \frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}+\frac{1}{r^2}\frac{\partial^2}{\partial\theta^2};\alpha_1=\frac{mc^2\omega^2\left(S+I\right)}{D_2};\\
\alpha_2 &= \frac{mc^2\omega^2}{D_2}\left(1-\frac{mc^2\omega^2SI}{D_2}\right);\alpha_1^*=\frac{S}{D_2};
\end{aligned}
$$

$$m = \frac{h_2\rho_2}{R_0}; \alpha_2^* = \frac{1}{D_2}\left(1 - \frac{mc^2\omega^2 SI}{D_2}\right); S = \frac{E_2 h_2^2}{G_2 R_0^2 k_{T2}\left(1-\nu_{02}^2\right)};$$
$$I = \frac{h_2^2}{12R_0^2}; D_2 = \frac{E_2 h_2^3}{12R_0^2\left(1-\nu_{02}^2\right)};$$

$$k_3^2 = \frac{\varpi^2}{\beta_3^2} - \frac{2}{S\left(1-\nu_{02}\right)}; \beta_3^2 = \frac{c^2}{c_{22}^2}; c_{22}^2 = \frac{E_2 k_{T2}}{2\rho_2\left(1+\nu_{02}\right)};$$

g_z - normal component of outside force, which falls falls on middle surface area unit; $E_2, \nu_{02}, G_2, \rho_2, h_2$ is Young's modulus, Poisson's ratio, shift ratio, density and plate thickness appropriately, K_{T2} is numerical shift coefficient; Φ is auxiliary function; C_{22} is velocity of transversed waves in plate.

Let us define the radial and tangent components of motion vector of plates W_r, W_Θ which characterize the flat stress state as follows [2, 4]

$$W_z = \frac{\partial\varphi}{\partial r} + \frac{1}{r}\frac{\partial\Psi}{\partial r}; W_\theta = \frac{1}{r}\frac{\partial\varphi}{\partial\theta} - \frac{\partial\Psi}{\partial r}; \tag{7}$$

Here the scalar potential φ and non-zero component of vector potential ψ satisfy the wave equations

$$\left(\Delta_0 + \frac{\omega^2}{\beta_1^2}\right)\varphi(r,\theta) = 0; \left(\Delta_0 + \frac{\omega^2}{\beta_2^2}\right)\Psi(r,\theta) = 0 \tag{8}$$

where

$$\beta_1 = \frac{c_{11}}{c}; \beta_2 = \frac{c_{21}}{c}; c_{11}^2 = \frac{E_2\left(1-\nu_{02}\right)}{\rho\left(1+\nu_{02}\right)\left(1-2\nu_{02}\right)}; c_{21}^2 = \frac{E_2}{2\rho\left(1-\nu_{02}\right)}.$$

The solution of differential equations (2)-(8) must satisfy the following boundary conditions [2, 5]

a) continuity translation on middle cover surface and plates

$$\left[\frac{\partial}{\partial r}\left(P^a + P^e\right)\right]_{r=1} = \rho\omega^2 c^2 u_3; g_r = \left[-P^a - P^e + P^o\right]_{r=1} \tag{9}$$

$$\left[\frac{\partial}{\partial r}P^o\right]_{r=1} = \xi_{01}\beta_0^{-1}\rho\omega^2 c^2 u_3; \xi_{01} = \frac{\beta_0^2 h_1\rho_1}{R_0\rho};$$

$$\left[\frac{\partial}{\partial z}\left(P^a + P^e\right)\right]_{z=l} = \rho\omega^2 c^2 W_z; g_z = \left[-P^a - P^e + P^o\right]_{z=\pm l};$$

$$\left[\frac{\partial}{\partial z}P^o\right]_{z=l} = \xi_{01}\beta_0^{-1}\rho\omega^2 c^2 W_z; \xi_{02} = \frac{\beta_0^2 h_2\rho_2}{R_0\rho};$$

b) continuity translation in junction place of cover and plates

$$[u_3]_{z=l} = [W_r]_{r=1}; [u_5]_{z=l} = [W_z]_{r=1}; [u_1]_{z=l} = [W_\theta]_{r=1}; \tag{10}$$

c) continuity of turning angle ($r = 1, z = \pm l$)

$$[u_2]_{z=l} = \left[\frac{1}{R_0}\frac{\partial W_r}{\partial r} - \frac{W_\theta}{R_0}\right]_{r=1}; [u_4]_{z=l} = \left[-\frac{\partial G^*}{\partial r} + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right]_{r=1} \tag{11}$$

d) continuity of forces ($r = 1, z = l$)

$$B_1\left[\frac{\partial u_5}{\partial z} + \frac{\nu_{01}}{R_0}\left(\frac{\partial u_1}{\partial \theta} + u_3\right)\right] = \Lambda_1^{'}\left[\frac{\partial}{\partial r}\left(G^* - W_z\right) + \frac{1}{r}\frac{\partial \Phi}{\partial r}\right];$$

$$\Lambda_1^{'}\left(u_4 + \frac{\partial u_3}{\partial z}\right) = \frac{E_2 h_2}{\left(1-\nu_{02}^2\right)R_0}\left[\frac{\partial W_r}{\partial r} + \frac{\nu_{02}}{r}\left(W_z + \frac{\partial W_\theta}{\partial \theta}\right)\right]; \tag{12}$$

$$B_1\frac{1-\nu_{01}}{2}\left(\frac{1}{R_0}\frac{\partial u_5}{\partial \theta} + \frac{\partial u_1}{\partial z}\right) = \frac{E_2 h_2}{\left(1-\nu_{02}\right)R_0}\left(\frac{1}{r}\frac{\partial W_z}{\partial r} + \frac{\partial W_\theta}{\partial r} - \frac{W_\theta}{r}\right);$$

e) continuity of moments ($r = 1, z = l$)

$$D_1\left(\frac{\partial u_4}{\partial z} + \frac{\nu_{01}}{R_0}\frac{\partial u_2}{\partial \theta}\right) = -D_2\left[\frac{\partial^2 G^*}{\partial r^2} + \nu_{02}\left(\frac{1}{r}\frac{\partial G^*}{\partial r} + \frac{1}{r^2}\frac{\partial^2 G^*}{\partial r^2}\right) - \frac{1-\nu_{02}}{2}\frac{\partial \Phi}{\partial r}\right]; \tag{13}$$

$$D_1\frac{1-\nu_{01}}{2}\left(\frac{\partial u_2}{\partial z} + \frac{1}{R_0}\frac{\partial u_4}{\partial \theta} + \frac{\partial u_1}{\partial z}\right) =$$

$$-D_2\left(1-\nu_{02}\right)\left\{\frac{\partial^2 G^*}{\partial r \partial \theta} + \frac{1}{2}\left[\Delta_0 \Phi - 2\left(\frac{1}{r}\frac{\partial \Phi}{\partial r} + \frac{1}{r^2}\frac{\partial^2 \Phi}{\partial \theta^2}\right)\right]\right\}$$

where

$$B_1 = \frac{E_1 h_1}{\left(1-\nu_{02}^2\right)R_0}; G^* = W_z + S\Delta_0 W_z + \frac{S^2}{D_2}g_z;$$

$$\Lambda_\sigma^{'} = \frac{k_{t\sigma} h_\sigma}{R_0}; D_\sigma = \frac{E_\sigma h_\sigma^3}{12\left(1-\nu_{0\sigma}^2\right)R_0^3}; \sigma = 1, 2.$$

Taking into account the symmetry relative to plane $\Theta = 0$ we find the solution of equations (2)-(8) in the Fourier series form:

$$u_1(z,\theta) = \sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(u_{1n\nu}^{+}\cos\beta_\nu z + u_{1n\nu}^{-}\sin\beta_\nu z\right)\sin n\theta;$$

$$u_2(z,\theta) = \sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(u_{2n\nu}^{+}\cos\beta_\nu z + u_{2n\nu}^{-}\sin\beta_\nu z\right)\sin n\theta;$$

$$u_3(z,\theta) = \sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(u_{3n\nu}^{+}\cos\beta_\nu z + u_{3n\nu}^{-}\sin\beta_\nu z\right)\cos n\theta;$$

$$u_4(z,\theta) = \sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(u_{4n\nu}^{+}\sin\beta_\nu z + u_{4n\nu}^{-}\cos\beta_\nu z\right)\cos n\theta;$$

$$u_5(z,\theta)=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(u_{5n\nu}^{+}\sin\beta_\nu z+u_{5n\nu}^{-}\cos\beta_\nu z\right)\cos n\theta \tag{14}$$

$$W_z(r,\theta)=\sum_{n=0}^{\infty}W_{z,n}(r)\cos n\theta;W_r(r,\theta)=\sum_{n=0}^{\infty}W_{r,n}(r)\cos n\theta;$$

$$\varphi(r,\theta)=\sum_{n=0}^{\infty}\varphi_n(r)\cos n\theta;\Psi(r,\theta)=\sum_{n=0}^{\infty}\Psi_n(r)\sin n\theta$$

$$W_{\theta n}(r,\theta)=\sum_{n=0}^{\infty}W_{\theta,n}(r)\sin n\theta;\Phi(r,\theta)=\sum_{n=0}^{\infty}\Phi_n(r)\sin n\theta$$

$$P^e(r,z,\theta)=\sum_{n=0}^{\infty}P_n^e(r,z)\cos n\theta;P^0(r,z,\theta)=\sum_{n=0}^{\infty}P_n^0(r,z)\cos n\theta$$

where

$$\beta_\nu=\frac{\nu\pi}{l};l=\frac{L}{R_0};$$

In relations (14) "+" is the symmetric and "-" is the non-symmetric components of general problem solution.

Let us exemplify the solution of wave equation (2) which satisfies the Zommerfield's condition in the form of Helmholtz-Huygens integral [6]

$$P^e(r,z,\theta)=\int_{\sigma_0}\left(P^e\mid_{\sigma_0}\frac{\partial G}{\partial n_0}-\frac{\partial P^e}{\partial n}\mid_{\sigma_0}G\right)d\sigma_0 \tag{15}$$

where $d\sigma_0$ is element of dissipating surface area σ_0;

$G=(4\pi R^*)^{-1}\exp(-i\omega R^*)$ is the fundamental solution of Helmholtz equation; $R^{*2}=r^2+r_0^2-2rr_0\cos(\Theta-\Theta_0)+(z-z_0)^2$ is the distance between point of observation (r,z,Θ) and point (r_0,z_0,Θ_0) on the surface σ_0; $\frac{\partial}{\partial n_0}$ is the derivative with respect to outside normal of surface σ_0 in point (r,z,Θ); $P^e(r_0,z_0,\Theta_0)$ is the value of dissipated pressure on surface σ_0.

Let us arrange the defined representation of function G in cylindrical coordinates [6] with divided coordinate z

$$G=\frac{1}{4\pi}\sum_{m=0}^{\infty}\epsilon_m\cos\left[m(\theta-\theta_0)\right]\int_0^{\infty}\lambda J_m(\lambda r_0)J_m(\lambda r)\frac{e^{-\varkappa|z-z_0|}}{\varkappa}d\lambda \tag{16}$$

where æ=$\begin{cases}\sqrt{\lambda^2-\omega^2}, & \text{if } \lambda>\omega\\ i\sqrt{\omega^2-\lambda^2}, & \text{if } \omega>\lambda\end{cases}$, $J_m()$ is Bessel's function of the first kind; $\varepsilon_0=1;\varepsilon_m=2;m\geq 1$.

Let us assume on the surface of closed cylindrical cover the distributed pressure $P^e(r_0,z_0,\Theta_0)$ and its formal derivatives are presented in the following form:

a) by Fourier's series on the side surface of cylinder

$$P^e(z_0,\theta_0)=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(f_{n\nu}^{*+e}\cos\beta_\nu z_0+f_{n\nu}^{-e}\sin\beta_\nu z_0\right)\cos n\theta \tag{17}$$

$$\frac{\partial}{\partial r}P^e(z_0,\theta_0)=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}\left(f_{n\nu}^{*+e}\cos\beta_\nu z_0+f_{n\nu}^{*-e}\sin\beta_\nu z_0\right)\cos n\theta$$

b) by Fourier-Bessel's series on plates ends $z_0=\pm l$

$$P^e(z_0,\theta_0)=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}g_{nj}^{e\pm}J_n(\gamma_{jn}r_0)\cos n\theta_0$$

$$\frac{\partial}{\partial z}P^e(z_0,\theta_0)=\sum_{n=0}^{\infty}\sum_{\nu=0}^{\infty}g_{nj}^{*e\pm}J_n(\gamma_{jn}r_0)\cos n\theta_0 \tag{18}$$

where γ_{jn} is the zeroes of Bessel's function derivative ($J_n'(\theta_{jn})=0$)

$$g_{nj|z_0=l}^{e+}=g_{nj|z_0=-l}^{e+};g_{nj|z_0=l}^{e-}=g_{nj|z_0=-l}^{e-};g_{nj|z_0=l}^{*e+}=g_{nj|z_0=-l}^{*e+};g_{nj|z_0=l}^{*e-}=g_{nj|z_0=-l}^{*e-}$$

Let us write the distributed pressure P^e as the sum of symmetrical and antisymmetric components relative to $z=0$

$$P^e(r,z,\theta)=P^{e+}(r,z,\theta)+P^{e-}(r,z,\theta)$$

Substituting the expansions (16), (17), (18) into (15) and integrating using (14) we get the following expressions for the distributed pressure to region

a)symmetrical component

$$P_n^{e+}(r,z)=\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty}f_{n\nu}^{e+}\int_0^{\infty}\frac{\lambda^2 j_n'(\lambda)J_n(\lambda r)}{\varkappa^2+\beta_\nu^2}\left[\cos\beta_\nu z-(-1)^\nu ch(\varkappa z)e^{-\varkappa l}\right]d\lambda-$$

$$-\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty}f_{n\nu}^{*e+}\int_0^{\infty}\frac{\lambda j_n(\lambda)J_n(\lambda r)}{\varkappa^2+\beta_\nu^2}\left[\cos\beta_\nu z-(-1)^\nu ch(\varkappa z)e^{-\varkappa l}\right]d\lambda- \tag{19}$$

$$-\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty}g_{nj}^{e+}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n'(\lambda)J_n(\lambda r)}{\gamma_{jn}^2-\lambda^2}e^{-\varkappa l}sh(|\varkappa z|)d\lambda-$$

$$-\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty}g_{nj}^{*e+}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n'(\lambda)J_n(\lambda r)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)}e^{-\varkappa l}sh(|\varkappa z|)d\lambda.$$

b)antisymmetric component

$$P_n^{e-}(r,z)=\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty}f_{n\nu}^{e-}\int_0^{\infty}\frac{\lambda^2 j_n'(\lambda)J_n(\lambda r)}{\varkappa(\varkappa^2+\beta_\nu^2)}\left[\varkappa\sin\beta_\nu z-(-1)^\nu\beta_\nu sh(\varkappa z)e^{-\varkappa l}\right]d\lambda-$$

$$-\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty} f_{n\nu}^{*e-}\int_0^{\infty}\frac{\lambda J_n(\lambda)J_n(\lambda r)}{\varkappa(\varkappa^2+\beta_\nu^2)}\left[\varkappa\sin\beta_\nu z-(-1)^\nu\beta_\nu sh(\varkappa z)e^{-\varkappa l}\right]d\lambda\pm$$

$$\pm\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty} g_{nj}^{e-}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\gamma_{jn}^2-\lambda^2}e^{-\varkappa l}ch(\varkappa z)\,d\lambda\pm \tag{20}$$

$$\pm\frac{\varepsilon_n}{2}\sum_{\nu=0}^{\infty} g_{nj}^{*e-}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)}e^{-\varkappa l}sh\left(|\,\varkappa z\,|\right)d\lambda.$$

where $\varepsilon_0=1;\varepsilon_n=2;n\geq 1$.

For the region $(r\geq 0,|z|\geq l,0\leq\Theta\leq 2\pi)$ expression for $P_n^{e+}(r,z)$ and $P_n^{e-}(r,z)$ will look

$$P_n^{e+}(r,z)=\frac{\varepsilon_n}{2}\left\{\begin{array}{c}\sum_{\nu=0}^{\infty} f_{n\nu}^{e+}\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\varkappa^2+\beta_\nu^2}(-1)^\nu sh(\varkappa l)e^{-\varkappa|z|}d\lambda-\\ -\sum_{\nu=0}^{\infty} f_{n\nu}^{*e+}\int_0^{\infty}\frac{\lambda J_n(\lambda)J_n(\lambda r)}{\varkappa^2+\beta_\nu^2}(-1)^\nu sh(\varkappa l)e^{-\varkappa|z|}d\lambda+\\ +\sum_{j=0}^{\infty} g_{nj}^{e+}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{{\gamma_{jn}}^2-\lambda^2}sh(\varkappa l)e^{-\varkappa|z|}d\lambda-\\ -\sum_{j=0}^{\infty} g_{nj}^{*e+}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\varkappa({\gamma_{jn}}^2-\lambda^2)}ch(\varkappa l)e^{-\varkappa|z|}d\lambda\end{array}\right\} \tag{21}$$

$$P_n^{e-}(r,z)=\pm\frac{\varepsilon_n}{2}\left\{\begin{array}{c}-\sum_{\nu=0}^{\infty} f_{n\nu}^{e-}\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\varkappa^2+\beta_\nu^2}(-1)^\nu\beta_\nu sh(\varkappa l)e^{-\varkappa|z|}d\lambda-\\ -\sum_{\nu=0}^{\infty} f_{n\nu}^{*e-}\int_0^{\infty}\frac{\lambda J_n(\lambda)J_n(\lambda r)}{\varkappa(\varkappa^2+\beta_\nu^2)}(-1)^\nu\beta_\nu sh(\varkappa l)e^{-\varkappa|z|}d\lambda+\\ +\sum_{j=0}^{\infty} g_{nj}^{e-}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{{\gamma_{jn}}^2-\lambda^2}ch(\varkappa l)e^{-\varkappa|z|}d\lambda-\\ -\sum_{j=0}^{\infty} g_{nj}^{*e-}J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2 J_n^{|}(\lambda)J_n(\lambda r)}{\varkappa({\gamma_{jn}}^2-\lambda^2)}sh(\varkappa l)e^{-\varkappa|z|}d\lambda\end{array}\right\} \tag{22}$$

$+$ when $z>l$

$-$ when $z<-l$.

Functions $P_n^{e\pm}(r,z)$ and $P_n^{e\pm}(r_0,z_0)$ and introduced in series (17)-(18) unknown coefficients $f_{n\nu}^{e\pm},g_{nj}^{e\pm}$ and functions $P_n^{e\pm}(r_0,z_0)$ are tied by relations [7, 8, 9]

$$f_{n\nu}^{e+}=\frac{\varepsilon_\nu}{l}\int_{-j}^{l}P_n^{e+}(r,z)\mid_{r=1}\cos(\beta_\nu z)\,dz;\varepsilon_0=\frac{1}{2};\varepsilon_\nu=1,\nu\geq 1;$$

$$f_{n\nu}^{e-}=\frac{1}{l}\int_{-j}^{l}P_n^{e-}(r,z)\mid_{r=1}\sin(\beta_\nu z)\,dz; \tag{23}$$

$$g_{nj}^{e\pm}=\varepsilon_{nj}\int_0^1 P_n^{e\pm}(r,z)\mid_{z=l}J_n(\gamma_{jn}r)\,rdr;\epsilon_{nj}=\frac{2\gamma_{jn}^2}{\left(\gamma_{jn}^2-n^2\right)J_n^2(\gamma_{jn})}.$$

Substituting the expression (19),(20) into (23) we get algebraic equations

$$\sum_{\nu=0}^{\infty}\left(f_{n\nu}^{e\pm}F_{n\nu\mu}^{1\pm}+f_{n\nu}^{*e\pm}F_{n\nu\mu}^{*1\pm}\right)+\sum_{j=0}^{\infty}\left(g_{nj}^{e\pm}G_{njj\mu}^{1\pm}+g_{nj}^{*e\pm}G_{njj\mu}^{*1\pm}\right)=0;$$

$$\sum_{\nu=0}^{\infty}\left(f_{n\nu}^{e\pm}F_{n\nu\mu}^{2\pm}+f_{n\nu}^{*e\pm}F_{n\nu\mu}^{*2\pm}\right)+\sum_{j=0}^{\infty}\left(g_{nj}^{e\pm}G_{nj\mu}^{2\pm}+g_{nj}^{*e\pm}G_{nj\mu}^{*2\pm}\right)=0. \qquad (24)$$

where

$$F_{N\nu\mu}^{*1+}=-\int_0^{\infty}\frac{\lambda J_n^2(\lambda)}{(\varkappa^2+\beta_\nu^2)}\left[\varepsilon_\mu l\delta_{\nu\mu}-\frac{(-1)^{\nu+\mu}\varkappa\left(1-e^{-2\varkappa l}\right)}{(\varkappa^2+\beta_\nu^2)}\right]d\lambda;$$

$$F_{N\nu\mu}^{1+}=\int_0^{\infty}\frac{J_n^{'}(\lambda)J_n(\lambda)}{(\varkappa^2+\beta_\nu^2)}\left[\varepsilon_\mu l\delta_{\nu\mu}\left(\omega^2-\beta_\nu^2\right)-\frac{(-1)^{\nu+\mu}(y_1-y_2)}{\varkappa(\varkappa^2+\beta_\nu^2)}\right]d\lambda-$$

$$-\varepsilon_\mu l\delta_{\nu\mu}\left(\frac{n}{2}+1\right);y_1=\varkappa^2\lambda^2\left(1-e^{-2\varkappa l}\right);y_2=\left(\varkappa^2+\beta_\nu^2\right)^2;$$

$$G_{nj\mu}^{1+}=J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2J_n^{'}(\lambda)J_n(\lambda)(-1)^{\mu}\left(1-e^{-2\varkappa l}\right)}{\left(\gamma_{jn}^2-\lambda^2\right)\left(\varkappa^2+\beta_\mu^2\right)}d\lambda$$

$$G_{nj\mu}^{*1+}=-J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2J_n^{'}(\lambda)J_n(\lambda)(-1)^{\mu}\varkappa\left(1-e^{-2\varkappa l}\right)}{\left(\gamma_{jn}^2-\lambda^2\right)\left(\varkappa^2+\beta_\mu^2\right)}d\lambda;$$

$$G_{nj\mu}^{2+}=J_n(\gamma_{jn})J_n(\gamma_{\mu n})\int_0^{\infty}\frac{\lambda^3J_n^{'2}(\lambda)\left(1-e^{-2\varkappa l}\right)}{\left(\gamma_{jn}^2-\lambda^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda-\frac{\delta_{j\mu}}{\varepsilon_{n\mu}};$$

$$G_{nj\mu}^{*2+}=J_n(\gamma_{jn})J_n(\gamma_{\mu n})\int_0^{\infty}\frac{\lambda^3J_n^{'2}(\lambda)\left(1+e^{-2\varkappa l}\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda;$$

$$F_{n\nu\mu}^{2+}=J_n(\gamma_{\mu n})\int_0^{\infty}\frac{\lambda^3J_n^{'2}(\lambda)(-1)^{\nu}\left(1-e^{-2\varkappa l}\right)}{\left(\varkappa^2+\beta_\mu^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda;$$

$$F_{n\nu\mu}^{*2+}=J_n(\gamma_{\mu n})\int_0^{\infty}\frac{\lambda^2J_n^{'}(\lambda)J_n(\lambda)(-1)^{\nu}\left(1-e^{-2\varkappa l}\right)}{\left(\varkappa^2+\beta_\mu^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda;$$

$$F_{n\nu\mu}^{1-}=\int_0^{\infty}\frac{J_n^{'}(\lambda)J_n(\lambda)}{\varkappa(\varkappa^2+\beta_\nu^2)}\left[\varkappa l\delta_{\nu\mu}\left(\varpi^2-\beta_\nu^2\right)-\frac{(-1)^{\nu+\mu}\beta_\nu\beta_\mu\lambda^2\left(1-e^{-2\varkappa l}\right)}{\left(\varkappa^2+\beta_\mu^2\right)}\right]d\lambda-$$

$$-\delta_{\nu\mu}l\left(\frac{n}{2}+1\right);$$

$$F_{n\nu\mu}^{*1-}=\int_0^{\infty}\frac{\lambda J_n^2(\lambda)}{\varkappa(\varkappa^2+\beta_\nu^2)}\left[\varkappa l\delta_{\nu\mu}+\frac{(-1)^{\nu+\mu}\beta_\nu\left(1-e^{-2\varkappa l}\right)}{\left(\varkappa^2+\beta_\mu^2\right)}\right]d\lambda;$$

$$G_{nj\mu}^{1-}=J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2J_n^{'}(\lambda)J_n(\lambda)(-1)^{\mu}\beta_\mu\left(1-e^{-2\varkappa l}\right)}{\left(\gamma_{jn}^2-\lambda^2\right)\left(\varkappa^2+\beta_\mu^2\right)}d\lambda;$$

$$G_{nj\mu}^{*1-}=J_n(\gamma_{jn})\int_0^{\infty}\frac{\lambda^2J_n^{'}(\lambda)J_n(\lambda)(-1)^{\mu}\beta_\mu\left(1-e^{-2\varkappa l}\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)\left(\varkappa^2+\beta_\mu^2\right)}d\lambda;$$

$$F_{n\nu\mu}^{2-}=-J_n\left(\gamma_{\mu n}\right)\int_0^\infty\frac{\lambda^3J_n^{|}\left(\lambda\right)\left(-1\right)^\nu\beta_\nu\left(1-e^{-2\varkappa l}\right)}{\varkappa\left(\varkappa^2+\beta_\mu^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda;$$

$$F_{nj\mu}^{*2-}=-J_n\left(\gamma_{\mu n}\right)\int_0^\infty\frac{\lambda^2J_n^{|}\left(\lambda\right)J_n\left(\lambda\right)\left(-1\right)^\mu\beta_\nu\left(1-e^{-2\varkappa l}\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)\left(\varkappa^2+\beta_\mu^2\right)}d\lambda;$$

$$G_{nj\mu}^{2-}=J_n\left(\gamma_{jn}\right)J_n\left(\gamma_{\mu n}\right)\int_0^\infty\frac{\lambda^3J_n^{|2}\left(\lambda\right)\left(1+e^{-2\varkappa l}\right)}{\left(\gamma_{jn}^2-\lambda^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda-\frac{\delta_{j\mu}}{\varepsilon_{n\mu}};$$

$$G_{nj\mu}^{*2-}=J_n\left(\gamma_{jn}\right)J_n\left(\gamma_{\mu n}\right)\int_0^\infty\frac{\lambda^3J_n^{|2}\left(\lambda\right)\left(1-e^{-2\varkappa l}\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)\left(\gamma_{\mu n}^2-\lambda^2\right)}d\lambda.$$

$\delta_{\nu\mu}$ is the Kronecker's symbol.

Let us consider a question of derivatives representation from series (14) also by Fourier and Fourier-Bessel series, as during the solution of equations (3)-(8) we need to differentiate it.

Let us assume that even function $f\left(z\right)$ and odd function $\varphi\left(z\right)$ and its derivatives could be expanded into Fourier series in $\left[-l,l\right]$:

$$f^{(2k)}\left(z\right)=\sum_{\nu=0}^\infty a_\nu^{2k}\cos\beta_\nu z;\varphi^{(2k)}\left(z\right)=\sum_{\nu=1}^\infty b_\nu^{2k}\sin\beta_\nu z;$$

$$f^{(2k+1)}\left(z\right)=\sum_{\nu=0}^\infty a_\nu^{2k+1}\sin\beta_\nu z;\varphi^{(2k-1)}\left(z\right)=\sum_{\nu=1}^\infty b_\nu^{2k-1}\cos\beta_\nu z. \tag{25}$$

Coefficients $a_\nu^{2K},a_\nu^{2K-1},b_\nu^{2K},b_\nu^{2K-1}$ are defined by formulas [10, 11, 12]

$$a_\nu^{2k}=\frac{\varepsilon_\nu}{2}\int_{-l}^l f^{(2k)}\left(z\right)\cos\beta_\nu z dz;b_\nu^{2k}=\frac{\varepsilon_\nu}{2}\int_{-l}^l\varphi^{(2k)}\left(z\right)\sin\beta_\nu z dz$$

$$a_\nu^{2k+1}=\frac{\varepsilon_\nu}{2}\int_{-l}^l f^{(2k+1)}\left(z\right)\sin\beta_\nu z dz;b_\nu^{2k-1}=\frac{\varepsilon_\nu}{2}\int_{-l}^l\varphi^{(2k-1)}\left(z\right)\cos\beta_\nu z dz \tag{26}$$

Integrating (26) by parts and taking into account the evenness and oddness of derivatives from $f\left(z\right)$ and $\varphi\left(z\right)$ we find that

$$a_\nu^{2k}=\frac{2\varepsilon_\nu}{l}\sum\left[\left(-1\right)^{i+\nu+1}\beta_\nu^{2i-2}f^{(2k-2i+1)}\left(l\right)+\left(-1\right)^k\beta_\nu^{2k}a_\nu^0\right];$$

$$b_\nu^{2k-1}=\frac{2}{l}\sum\left[\left(-1\right)^{i+\nu+1}\beta_\nu^{2i-2}\varphi^{(2k-2i)}\left(l\right)-\left(-1\right)^k\beta_\nu^{2k-1}b_\nu^0\right]; \tag{27}$$

$$a_\nu^{2k+1}=\beta_\nu a_\nu^{2k};b_\nu^{2k}=-\beta_\nu b_\nu^{2k-1};$$

During the solution of equations (3) we present the function $P_n^0\left(r,z\right)$ as the series

$$\sum_{\nu=0}^\infty\left[P_{n\nu}^{o+}\left(r\right)\cos\beta_\nu z+P_{n\nu}^{o-}\left(r\right)\sin\beta_\nu z\right]=P_n^o\left(r,z\right) \tag{28}$$

The second derivative from $P_n^0(r,z)$ taking into account (25), (27) we write as

$$\frac{\partial^2}{\partial z^2}P_n^o(r,z)=\sum_{\nu=0}^{\infty}\left(\frac{\partial}{\partial z}P_n^{o+}(r,z)\mid_{z=l}\frac{2\varepsilon_\nu(-1)^\nu}{l}-\beta_\nu^2P_{n\nu}^{o+}(r)\right)\cos\beta_\nu z+$$

$$+\sum_{\nu=0}^{\infty}\left(-\frac{2(-1)^\nu\beta_\nu}{l}P_n^{o+}(r,z)\mid_{z=l}-\beta_\nu^2P_{n\nu}^{o-}(r)\right)\sin\beta_\nu z \tag{29}$$

Let the dissipated pressure and its derivatives on the interior side of the plates are defined as follows

$$P_n^{o\pm}(r,l)=\sum_{j=0}^{\infty}g_{nj}^{o\pm}J_n(\gamma_{jn}r);\frac{\partial}{\partial z}P_n^{o\pm}(r,l)=\sum_{j=0}^{\infty}g_{nj}^{*o\pm}J_n(\gamma_{jn}r) \tag{30}$$

Taking into account (18)-(30) wave equation (3) for the symmetrical and anti-symmetric components obtains the following form

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\left(\frac{n^2}{r^2}-\varkappa_2^2\right)\right]P_{n\nu}^{o+}(r)=-\sum_{j=0}^{\infty}\frac{2\varepsilon_\nu(-1)^\nu}{l}g_{nj}^{*o+}J_n(\gamma_{jn}r); \tag{31}$$

$$\left[\frac{\partial^2}{\partial r^2}+\frac{1}{r}\frac{\partial}{\partial r}-\left(\frac{n^2}{r^2}-\varkappa_2^2\right)\right]P_{n\nu}^{o-}(r)=\sum_{j=0}^{\infty}\frac{2\beta_\nu(-1)^\nu}{l}g_{nj}^{o-}J_n(\gamma_{jn}r);$$

where $æ_2^2=\frac{\omega^2}{\beta_0^2-\beta_\nu^2}$;

General solutions of (31) are

$$P_{n\nu}^{o+}(r)=f_{n\nu}^{o+}J_n(\varkappa_2r)-\sum_{j=0}^{\infty}C_{n\nu j}^{+}g_{nj}^{*o+}J_n(\gamma_{jn}r); \tag{32}$$

$$P_{n\nu}^{o-}(r)=f_{n\nu}^{o-}J_n(\varkappa_2r)+\sum_{j=0}^{\infty}C_{n\nu j}^{-}g_{nj}^{o-}J_n(\gamma_{jn}r);$$

where

$$C_{n\nu j}^{+}=\frac{2\varepsilon_\nu(-1)^\nu}{l\left(\varkappa_2^2-\gamma_{jn}^2\right)};C_{n\nu j}^{-}=\frac{2\beta_\nu(-1)^\nu}{l\left(\varkappa_2^2-\gamma_{jn}^2\right)}$$

From (32) we obtain that pressure $P_n^{0\pm}(r)$ and $\frac{\partial}{\partial r}P_n^{0\pm}(r)$ on the interior surface of cylindrical cover ($z=1$) are

$$\begin{aligned}\frac{\partial}{\partial r}P_n^{o\pm}(1) &= f_{n\nu}^{o\pm}(\varkappa_2)J_n^{|}(\varkappa_2). \\ P_{n\nu}^{o+}(1) &= f_{n\nu}^{o+}J_n(\varkappa_2)-\sum_{j=0}^{\infty}C_{n\nu j}^{+}g_{nj}^{*o+}J_n(\gamma_{jn})\end{aligned} \tag{33}$$

$$P_{n\nu}^{o-}(1) = f_{n\nu}^{o-} J_n(\varkappa_2) + \sum_{j=0}^{\infty} C_{n\nu j}^{-} g_{nj}^{o-} J_n(\gamma_{jn})$$

Let us exemplify $P_{n\nu}^{0\pm}$ from (32) by Fourier-Bessel's series and equate it to coefficients (30). We get

$$g_{nj}^{o-} = \sum_{\nu=0}^{\infty} f_{n\nu}^{o+} \frac{(-1)^{\nu} \varkappa_2 J_n'(\varkappa_2) J_n(\gamma_{jn}) \varepsilon_{n\mu}}{\gamma_{jn}^2 - \varkappa_2^2} - g_{nj}^{*o+} \sum_{\nu=0}^{\infty} (-1)^{\nu} C_{n\nu\mu}^{+}; \qquad (34)$$

$$g_{nj}^{*o-} = \sum_{\nu=0}^{\infty} f_{n\nu}^{o+} \frac{(-1)^{\nu} \varkappa_2 J_n'(\varkappa_2) J_n(\gamma_{jn}) \varepsilon_{n\mu}}{\gamma_{jn}^2 - \varkappa_2^2} - g_{nj}^{o+} \sum_{\nu=0}^{\infty} (-1)^{\nu} C_{n\nu\mu}^{-}. \qquad (35)$$

Let us represent the falling pressure $P^a(r, z, \Theta)$ and ints normal derivatives on the surface of a cover and plates a the series analogical to (14), (18), (19):

$$P^a(r, z, \theta) = \sum_{n=0}^{\infty} P_n^a(r, z) \cos n\theta;$$

$$P_n^a(1, z) = \sum_{\nu=0}^{\infty} \left(f_{n\nu}^{a+} \cos \beta_\nu z + f_{n\nu}^{a-} \sin \beta_\nu z \right); P_n^{a\pm z}(r, l) = \sum_{j=0}^{\infty} g_{nj}^{a\pm} J_n(\gamma_{jn} r);$$

$$\frac{\partial}{\partial r} P_n^a(r, z) \mid_{r=1} = \sum_{\nu=0}^{\infty} \left(f_{n\nu}^{*a+} \cos \beta_\nu z + f_{n\nu}^{*a-} \sin \beta_\nu z \right); \qquad (36)$$

$$\frac{\partial}{\partial z} P_n^{a\pm}(r, l) \mid_{z=\pm j} = \sum_{j=0}^{\infty} \left(g_{nj}^{*a+} \pm g_{nj}^{*a+} \right) J_n(\gamma_{jn} r);$$

Let us present the flat wave (1) as the expansion [6, 15]

$$P^a(r, z, \theta) = P_0 \sum_{j=0}^{\infty} \varepsilon_n i^{-n} J_n(\omega \sin \varphi^* r) \exp(-i\omega z \cos \varphi^*) \cos n\theta;$$

and integrating using formulas that define the correspondent series components (36) we obtain

$$f_{n\nu}^{a+} = -\frac{(-1)^{\nu} 2\varepsilon_n i^{-n} J_n(\omega \sin \varphi^*) \omega \cos \varphi^*}{\beta_\nu^2 - \omega^2 \cos^2 \varphi^*} \sin(\omega l \cos \varphi^*);$$

$$f_{n\nu}^{a-} = \frac{2(-1)^{\nu} \varepsilon_n i^{-n} J_n(\omega \sin \varphi^*) \beta_\nu}{\beta_\nu^2 - \omega^2 \cos^2 \varphi^*} \sin(\omega l \cos \varphi^*);$$

$$g_{n\nu}^{a-} = -\frac{\varepsilon_{nj} \varepsilon_n i^{-n} \omega \sin \varphi^* J_n'(\omega \sin \varphi^*) J_n(\gamma_{jn})}{\gamma_{jn}^2 - \omega^2 \sin^2 \varphi^*} \cos(\omega l \cos \varphi^*);$$

$$g_{n\nu}^{a+} = \frac{\varepsilon_n i^{-n} \varepsilon_{nj} (\omega \sin \varphi^*) J_n(\gamma_{jn})}{\gamma_{jn}^2 - \omega^2 \sin^2 \varphi^*} \sin(\omega l \cos \varphi^*);$$

$$f_{n\nu}^{*a+}=-\varepsilon_n i^{-n}\omega\sin\varphi^* J_n^{'}\left(\omega\sin\varphi^*\right)\frac{2\cos\varphi^*\left(-1\right)^{\nu}\omega}{\beta_\nu^2-\omega^2\cos^2\varphi^*}\sin\left(\omega l\cos\varphi^*\right);$$

$$f_{n\nu}^{*a-}=\varepsilon_n i^{-n}\omega\sin\varphi^* J_n^{'}\left(\omega\sin\varphi^*\right)\frac{2\beta_\nu}{\beta_\nu^2-\omega^2\cos^2\varphi^*}\sin\left(\omega l\cos\varphi^*\right);$$

$$g_{n\nu}^{*a+}=i\omega\cos\varphi^* g_{nj}^{a+}; g_{n\nu}^{*a-}=i\omega\cos\varphi^* g_{nj}^{a-}.$$

To find the solution of inhomogeneous equation (5) we define the auxiliary function

$$F^{\pm}=\Delta_0 W^{\pm}+\frac{S}{D_2}g_z^{\pm}; \tag{37}$$

Taking into account (37) to define $F^{\pm}\left(r,\Theta\right)$ from (5) we get the following differential equation:

$$\left(\Delta_0^2-\alpha_1\Delta_0-\alpha_2\right)F^{\pm}\left(r,\theta\right)=\left(\beta_1^*-\beta_2^*\right)g_z^{\pm}\left(r,z\right); \tag{38}$$

where

$$\beta_1^*=\alpha_2^*+\alpha_1^*\frac{S}{D_2}; \beta_2^*=\alpha_2\frac{S}{D_2};$$

$$g_z^{\pm}\left(r,z\right)=\sum_{n=0}^{\infty}\sum_{j=0}^{\infty}\left(g_{nj}^{a\pm}+g_{nj}^{e\pm}-g_{nj}^{o\pm}\right)J_n\left(\gamma_{jn}r\right)\cos n\theta;$$

The solution of equation (38) we write as follows:

$$F^{\pm}\left(r,\theta\right)=\sum_{n=0}^{\infty}F_n^{\pm}\left(r\right)\cos n\theta$$

$$F_n^{\pm}\left(r\right)=A_{1n}^{\pm}J_n\left(\gamma_1 r\right)+A_{2n}^{\pm}J_n\left(\gamma_2 r\right)+\sum_{j=0}^{\infty}A_{3nj}g_{znj}J_n\left(\gamma_{jn}r\right) \tag{39}$$

$$\gamma_{1,2}=\sqrt{\frac{\alpha_1\pm\sqrt{\alpha_1^2+4\alpha_2}}{2}}; A_{3nj}=\frac{-\beta_1^*\gamma_{jn}^2-\beta_2^*}{\gamma_{jn}^4-\alpha_1\gamma_{jn}^2-\alpha_2};$$

$$g_{znj}^{\pm}=g_{nj}^{a\pm}+g_{nj}^{e\pm}+g_{nj}^{o.\pm}.$$

$A_{1n}^{\pm}, A_{2n}^{\pm}$ are unknown coefficients.

Using (39) and (37) to the normal shifts of the plates we obtain

$$W_n^{\pm}\left(r\right)=\frac{A_{1n}^{\pm}J_n\left(\gamma_1 r\right)}{-\gamma_1^2}+\frac{A_{2n}^{\pm}J_n\left(\gamma_2 r\right)}{-\gamma_2^2}+\sum_{j=0}^{\infty}\frac{\left(A_{3nj}-\frac{S}{D_2}\right)}{-\gamma_{jn}^2}g_{znj}^{\pm}J_n\left(\gamma_{jn}r\right). \tag{40}$$

The solution of equations (6)-(8) will be rewritten as follows:

$$\Phi_n^{\pm}\left(r\right)=\Phi_{1n}^{\pm}J_n\left(k_1 r\right); \tag{41}$$

$$\varphi_n^{\pm}\left(r\right)=\varphi_{1n}^{\pm}J_n\left(k_1 r\right) \tag{42}$$

$$\Psi_n^{\pm}(r) = \Psi_{1n}^{\pm} J_n(k_2 r) \tag{43}$$

Substituting expansions (14) into equation (4) taking into account (25), (27) we get the algebraic equations relatively to components of shift vector components:

$$a_{ij}^{\pm} u_{nj\nu} = b_{in}^{\pm} + \delta_{i3} g_{n\nu}^{\pm}; a_{ij}^{\pm} = a_{ji}^{\pm}; i,j = 1,2,3,4,5. \tag{44}$$

where

$$a_{11}^{\pm} = \left(1+a^2\right)\left(-n^2 - \sigma_1\beta_\nu^2 + \varkappa_1^2\right) - \sigma_1\beta_\nu^2 - \frac{\omega^2}{\beta_\nu^2};$$

$$a_{12}^{\pm} = a^2\left(n^2 - \sigma_1\beta_\nu^2 + \varkappa_1^2\right) - \frac{\omega^2}{\beta_\nu^2}; a_{13}^{\pm} = \left(1+a^2\right)\left(1+\varkappa_1^2\right)(\pm n);$$

$$a_{14}^{\pm} = \beta_\nu\sigma_2\left(\begin{smallmatrix}-\\+\end{smallmatrix} n\right); a_{15}^{\pm} = 0; a_{22}^{\pm} = a^2\left(-n^2 - \sigma_1\beta_\nu^2 - \varkappa_1^2 + \frac{\omega^2}{\beta_\nu^2}\right) - \varkappa_1^2;$$

$$a_{23}^{\pm} = \left[\left(1+a^2\right)\varkappa_1^2 + a^2\right](\pm n); a_{24}^{\pm} = 0; a_{25}^{\pm} = a^2\sigma_2\beta_\nu^2(\pm n);$$

$$a_{34}^{\pm} = \nu_{01}\beta_\nu; a_{33}^{\pm} = \left(1+a^2\right)\left(1+\varkappa_1^2 n^2\right) + \varkappa_1^2\beta_\nu^2 - \frac{\omega^2}{\beta_\nu^2}; a_{35}^{\pm} = \begin{smallmatrix}-\\+\end{smallmatrix}\beta_\nu\varkappa_1^2$$

$$a_{44}^{\pm} = -\left(1+a^2\right)\sigma_1 n^2 - \beta_\nu^2 + \frac{\omega^2}{\beta_\nu^2}; a_{45}^{\pm} = a^2\left(n^2\sigma_1 - \beta_\nu^2 + \frac{\omega^2}{\beta_\nu^2}\right);$$

$$a_{55}^{\pm} = a^2\left(-n^2\sigma_1 - \beta_\nu^2 + \frac{\omega^2}{\beta_\nu^2}\right);$$

$$b_{1n}^{+} = C_{11}^{+}u_{1n}^{\prime+} + C_{12}^{+}u_{2n}^{\prime+} + C_{14}^{+}u_{4n}^{+}; b_{2n}^{+} = C_{21}^{+}u_{1n}^{\prime+} + C_{22}^{+}u_{2n}^{\prime+} + C_{25}^{+}u_{5n}^{+};$$

$$b_{3n}^{+} = C_{33}^{+}u_{3n}^{\prime+} + C_{34}^{+}u_{4n}^{+} + C_{35}^{+}u_{5n}^{+} + g_{rn\nu}^{+}; b_{4n}^{+} = C_{44}^{+}u_{4n}^{+} + C_{45}^{+}u_{5n}^{+};$$

$$b_{5n}^{+} = C_{54}^{+}u_{4n}^{+} + C_{55}^{+}u_{5n}^{+}; b_{1n}^{-} = C_{11}^{-}u_{1n}^{-} + C_{12}^{-}u_{2n}^{-} + C_{14}^{-}u_{4n}^{-};$$

$$b_{2n}^{-} = C_{21}^{-}u_{1n}^{-} + C_{22}^{-}u_{2n}^{-} + C_{25}^{-}u_{5n}^{-}; b_{3n}^{-} = C_{33}^{-}u_{3n}^{-} + g_{rn\nu}^{-};$$

$$b_{4n}^{-} = C_{41}^{-}u_{1n}^{-} + C_{43}^{-}u_{3n}^{-} + C_{44}^{-}u_{4n}^{\prime-} + C_{45}^{-}u_{5n}^{\prime-}; g_{rn\nu}^{-} = f_{n\nu}^{e+} + f_{n\nu}^{a+} - f_{n\nu}^{o+};$$

$$b_{5n}^{-} = C_{52}^{-}u_{2n}^{-} + C_{53}^{-}u_{3n}^{-} + C_{54}^{-}u_{4n}^{\prime-} + C_{55}^{-}u_{5n}^{\prime-}; C_{11}^{+} = -\sigma_1\frac{2\varepsilon_\nu(-1)^\nu}{l};$$

$$C_{12}^{+} = -a^2\sigma_1\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{14}^{+} = \sigma_2 n\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{21}^{+} = C_{12}^{+};$$

$$C_{22}^{+} = -a^2\sigma_2\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{25}^{+} = a^2\sigma_2 n\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{33}^{+} = \varkappa_1^2\frac{2\varepsilon_\nu(-1)^\nu}{l};$$

$$C_{34}^{+} = -\nu_{01}\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{35}^{+} = C_{33}^{+}; C_{44}^{+} = \beta_\nu\frac{2\varepsilon_\nu(-1)^\nu}{l}; C_{45}^{+} = a^2\beta_\nu\frac{2\varepsilon_\nu(-1)^\nu}{l};$$

$$C_{54}^{+} = C_{55}^{+} + C_{45}^{+}; C_{11}^{-} = \sigma_1\frac{2(-1)^\nu\beta_\nu}{l}; C_{12}^{-} = a^2\frac{2(-1)^\nu\beta_\nu}{l}; C_{21}^{-} = C_{12}^{-};$$

$$C_{33}^{-} = -\varkappa_1^2\frac{2(-1)^\nu\beta_\nu}{l}; C_{33}^{-} = -\sigma_2 h\frac{2(-1)^\nu\beta_\nu}{l}; C_{53}^{-} = C_{33}^{-}; C_{54}^{-} = C_{55}^{-} = C_{45}^{-};$$

$$C_{52}^{-}=a^{2}C_{41}^{-};C_{45}^{\pm}=\pm C_{12}^{\pm};$$

From (44) we define $U_{jn\nu}^{\pm}$ as:

$$u_{nj\nu}^{+}=L_{1n\nu}^{j+}u_{1n}^{\prime+}+L_{2n\nu}^{j+}u_{2n}^{\prime+}+L_{3n\nu}^{j+}u_{3n}^{\prime+}+L_{4n\nu}^{j+}u_{4n}^{+}+L_{5n\nu}^{j+}u_{5n}^{+}+L_{6n\nu}^{j+}g_{nr\nu}^{+};$$

$$u_{nj\nu}^{-}=L_{1n\nu}^{j-}u_{1n}^{-}+L_{2n\nu}^{j-}u_{2n}^{-}+L_{3n\nu}^{j-}u_{3n}^{-}+L_{4n\nu}^{j-}u_{4n}^{\prime-}+L_{5n\nu}^{j-}u_{5n}^{\prime-}+L_{6n\nu}^{j-}g_{nr\nu}^{-};j=\overline{1,5};\tag{45}$$

where

$$L_{1n\nu}^{j+}=\frac{1}{\Delta_{5}^{+}}\left(C_{11}^{+}A_{1j}^{+}+C_{21}^{+}A_{2j}^{+}\right);L_{2n\nu}^{j+}=\frac{1}{\Delta_{5}^{+}}\left(C_{12}^{+}A_{1j}^{+}+C_{22}^{+}A_{2j}^{+}\right);$$

$$L_{3n\nu}^{j+}=\frac{1}{\Delta_{5}^{+}}\left(C_{33}^{+}A_{3j}^{+}\right);L_{6n\nu}^{j+}=\frac{A_{3j}^{+}}{\Delta_{5}^{+}};$$

$$L_{4n\nu}^{j+}=\frac{1}{\Delta_{5}^{+}}\left(C^{+}{}_{44}A_{4j}^{+}+C_{54}^{+}A_{5j}^{+}+C^{+}{}_{14}A_{1j}^{+}+C_{34}^{+}A_{3j}^{+}\right);$$

$$L_{5n\nu}^{j+}=\frac{1}{\Delta_{5}^{+}}\left(C^{+}{}_{45}A_{4j}^{+}+C_{55}^{+}A_{5j}^{+}+C^{+}{}_{25}A_{2j}^{+}+C_{35}^{+}A_{3j}^{+}\right);$$

$$L_{1n\nu}^{j-}=\frac{1}{\Delta_{5}^{-}}\left(C^{-}{}_{11}A_{1j}^{-}+C_{21}^{-}A_{2j}^{-}+C^{-}{}_{41}A_{2j}^{-}\right);$$

$$L_{2n\nu}^{j-}=\frac{1}{\Delta_{5}^{-}}\left(C^{-}{}_{12}A_{1j}^{-}+C_{22}^{-}A_{2j}^{-}+C^{-}{}_{52}A_{5j}^{-}\right);$$

$$L_{3n\nu}^{j-}=\frac{1}{\Delta_{5}^{-}}\left(C^{-}{}_{33}A_{3j}^{-}+C_{43}^{-}A_{4j}^{-}+C^{-}{}_{53}A_{5j}^{-}\right);$$

$$L_{4n\nu}^{j-}=\frac{1}{\Delta_{5}^{-}}\left(C^{-}{}_{44}A_{4j}^{-}+C_{54}^{-}A_{5j}^{-}\right);L_{5n\nu}^{j-}=\frac{1}{\Delta_{5}^{-}}\left(C^{-}{}_{45}A_{4j}^{-}+C_{55}^{-}A_{5j}^{-}\right);$$

$$L_{6n\nu}^{j-}=\frac{A_{3j}^{-}}{\Delta_{5}^{-}};\Delta_{5}^{\pm}=\det\left|a_{ij}^{\pm}\right|;i,j=(1,2,3,4,5);$$

$A_{i,j}^{\pm}$ are algebraic adjuncts of elements $a_{ij}^{\pm}$.

To improve the convergence of the obtained solution (45) we use the asymptotic properties of expansion coefficients of Fourier series of the desired functions. Basing on (25), (27) we could define the following relations to the even functions $f(z)$ and odd functions $\varphi(z)$ and its derivatives represented as Fourier series on $[-l,l]$,

a) for even function $f(z)$

$$f(z)=\sum_{\nu=0}^{N}f_{\nu}\cos\beta_{\nu}z+\sum_{\nu=N+1}^{\infty}\frac{2\varepsilon_{\nu}(-1)^{\nu}}{l}\left[\frac{f^{\prime}(l)}{\beta_{\nu}^{2}}-\frac{f^{\prime\prime\prime}(l)}{\beta_{\nu}^{4}}+\frac{f^{\mathrm{v}}(l)}{\beta_{\nu}^{6}}\right]\cos\beta_{\nu}z;\tag{46}$$

$$\frac{\partial^2 f(z)}{\partial z^2}=\sum_{\nu=0}^{N}\left[-\beta_\nu f_\nu+\frac{2\varepsilon_\nu(-1)^\nu}{l}f^{\prime}(l)\right]\cos\beta_\nu z+\sum_{\nu=N+1}^{\infty}\frac{2\varepsilon_\nu(-1)^\nu}{l}\left[\frac{f^{\prime\prime\prime}(l)}{\beta_\nu^4}-\frac{f^{\mathrm{v}}(l)}{\beta_\nu^6}\right]\cos\beta_\nu z$$

$$\frac{\partial^4 f(z)}{\partial z^4}=\sum_{\nu=0}^{N}\left[\beta_\nu^4 f_\nu+\frac{2\varepsilon_\nu(-1)^\nu}{l}\left(\beta_\nu^2 f^{\prime}(l)-f^{\prime\prime\prime}(l)\right)\right]\cos\beta_\nu z+$$

$$+\sum_{\nu=N+1}^{\infty}\frac{2\varepsilon_\nu(-1)^\nu}{l}\left[\frac{f^{\mathrm{v}}(l)}{\beta_\nu^2}-\frac{f^{\mathrm{vii}}(l)}{\beta_\nu^6}\right]\cos\beta_\nu z;\ f^{\prime\prime\prime}(l)=\frac{\partial^{\prime} f(z)}{\partial z}\mid_{z=l};$$

$$f^{\prime\prime\prime}(l)=\frac{\partial^3 f(z)}{\partial z^3}\mid_{z=l};f^{\mathrm{v}}(l)=\frac{\partial^5 f(z)}{\partial z^5}\mid_{z=l};f^{\mathrm{vii}}(l)=\frac{\partial^7 f(z)}{\partial z^7}\mid_{z=l};$$

etc.

b) for odd function

$$\varphi(z)=\sum_{\nu=0}^{N}\varphi_\nu\sin\beta_\nu z;$$

$$\frac{\partial\varphi(z)}{\partial z}=\sum_{\nu=1}^{N}\left[\beta_\nu\varphi_\nu+\frac{2(-1)^\nu}{l}\varphi(l)\right]\cos\beta_\nu z+\sum_{\nu=N+1}^{\infty}\frac{2(-1)^\nu}{l}\left[\frac{\varphi^{\prime\prime}(l)}{\beta_\nu}-\frac{\varphi^{\mathrm{iv}}(l)}{\beta_\nu^4}\right]\cos\beta_\nu z \tag{47}$$

$$\frac{\partial^3\varphi(z)}{\partial z^3}=\sum_{\nu=1}^{N}\left[-\beta_\nu^3\varphi_\nu-\frac{2(-1)^\nu}{l}\left(\beta_\nu^2\varphi(l)-\varphi^{\prime\prime}(l)\right)\right]\cos\beta_\nu z+$$

$$+\sum_{\nu=N+1}^{\infty}\frac{2(-1)^\nu}{l}\left[\frac{\varphi^{\mathrm{iv}}(l)}{\beta_\nu^2}-\frac{\varphi^{\mathrm{vi}}(l)}{\beta_\nu^4}\right]\cos\beta_\nu z;\frac{\partial^2\varphi(z)}{\partial z^2}\mid_{z=l}=\varphi^{\prime\prime}(l);$$

$$\frac{\partial^5\varphi(z)}{\partial z^5}=\sum_{\nu=1}^{N}\left[\beta_\nu^5\varphi_\nu+\frac{2(-1)^\nu}{l}\left(\beta_\nu^4\varphi(l)-\beta_\nu\varphi^{\prime\prime}(l)\right)\right]\cos\beta_\nu z+$$

$$+\sum_{\nu=N+1}^{\infty}\frac{2(-1)^\nu}{l}\left[\frac{\varphi^{\mathrm{iv}}(l)}{\beta_\nu^2}-\frac{\varphi^{\mathrm{vi}}(l)}{\beta_\nu^4}\right]\cos\beta_\nu z;\frac{\partial^4\varphi(z)}{\partial z^4}\mid_{z=l}=\varphi^{\mathrm{iv}}(l);\frac{\partial^6\varphi(z)}{\partial z^6}\mid_{z=l}=\varphi^{\mathrm{vi}}(l).$$

Let us introduce the symbolism

$$X_1^+=u_{1n}^{\prime+};X_2^+=u_{2n}^{\prime+};X_3^+=u_{3n}^{\prime+};X_4^+=u_{4n}^{+};X_5^+=u_{5n}^{+};$$

$$X_1^-=u_{1n}^{-};X_2^-=u_{2n}^{-};X_3^-=u_{3n}^{-};X_4^-=u_{4n}^{\prime-};X_5^-=u_{5n}^{\prime-}; \tag{48}$$
$$X_6^\pm=A_{1n}^\pm;X_7^\pm=A_{2n}^\pm;X_8^\pm=\Phi_{1n}^\pm;X_9^\pm=\varphi_{1n}^\pm;X_{10}^\pm=\Psi_{1n}^+.$$

Substituting obtained solutions of differential equations (2)-(8) into the boundary conditions (9)-(13) we get the system of equations relative to unknown constants (48) with asymptotic (46), (47) for $U_{jn\nu}^\pm$ taken into account:

$$\left(a^\pm\right)_{ij}\left\{X^\pm\right\}_i=\left\{F^\pm\right\}_i; \tag{49}$$

where

$$a_{11}^{+}=\sum_{\nu=0}^{N}L_{1n\nu}^{3+}\left(-1\right)^{\nu};a_{12}^{+}=\sum_{\nu=0}^{N}L_{2n\nu}^{3+}\left(-1\right)^{\nu};a_{13}^{+}=\sum_{\nu=0}^{N}L_{3n\nu}^{3+}\left(-1\right)^{\nu}+B_{1}^{*};a_{14}^{+}=\sum_{\nu=0}^{N}L_{4n\nu}^{3+}\left(-1\right);$$

$$a_{15}^{+}=\sum_{\nu=0}^{N}L_{5n\nu}^{3+}\left(-1\right)^{\nu};a_{25}^{+}=1;a_{19}^{\pm}=-k_{1}J_{n}^{\shortmid}\left(k_{1}\right);a_{110}^{\pm}=-nJ_{n}\left(k_{2}\right);B_{1}^{*}=\frac{2l}{\pi}\sum_{\nu=N+1}^{\infty}\frac{1}{\nu^{2}};$$

$$a_{31}^{+}=\sum_{\nu=0}^{N}L_{1n\nu}^{1+}\left(-1\right)^{\nu}+B_{1}^{*};a_{32}^{+}=\sum_{\nu=0}^{N}L_{2n\nu}^{1+}\left(-1\right)^{\nu};a_{33}^{+}=\sum_{\nu=0}^{N}L_{3n\nu}^{1+}\left(-1\right)^{\nu};a_{34}^{+}=\sum_{\nu=0}^{N}L_{4n\nu}^{1+}\left(-1\right)^{\nu};$$

$$a_{26}^{\pm}=\frac{J_{n}\left(\gamma_{1}\right)}{\gamma_{1}};a_{27}^{\pm}=\frac{J_{n}\left(\gamma_{2}\right)}{\gamma_{2}};a_{310}^{\pm}=k_{2}J_{n}^{\shortmid}\left(k_{2}\right);a_{39}^{\pm}=-nJ_{n}\left(k_{1}\right);$$

$$a_{35}^{+}=\sum_{\nu=0}^{N}L_{5n\nu}^{1+}\left(-1\right)^{\nu};a_{41}^{+}=\sum_{\nu=0}^{N}L_{1n\nu}^{2+}\left(-1\right)^{\nu};a_{42}^{+}=\sum_{\nu=0}^{N}L_{2n\nu}^{2+}\left(-1\right)^{\nu}+B_{1}^{*};a_{43}^{+}=\sum_{\nu=0}^{N}L_{3n\nu}^{2+}\left(-1\right)^{\nu};$$

$$a_{44}^{+}=\sum_{\nu=0}^{N}L_{4n\nu}^{2+}\left(-1\right)^{\nu};a_{45}^{+}=\sum_{\nu=0}^{N}L_{3n\nu}^{2+}\left(-1\right)^{\nu};a_{54}^{+}=1;a_{49}^{\pm}=\frac{n}{R_{0}}\left(k_{1}J_{n}^{\shortmid}\left(k_{1}\right)-J_{n}\left(k_{1}\right)\right);$$

$$a_{410}^{\pm}=\frac{nJ_{n}\left(k_{2}\right)}{R_{0}}-\frac{k_{2}J_{n}^{\shortmid}\left(k_{2}\right)}{R_{0}};a_{56}^{\pm}=\left(-\frac{1}{\gamma_{1}}+S\gamma_{1}\right)J_{n}^{\shortmid}\left(\gamma_{1}\right);a_{57}^{\pm}=\left(-\frac{1}{\gamma_{2}}+S\gamma_{2}\right)J_{n}^{\shortmid}\left(\gamma_{2}\right);$$

$$a_{61}^{+}=B_{1}\left\{\sum_{\nu=1}^{N}L_{1n\nu}^{1+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{\nu_{01}}{R_{0}}n\left[\sum_{\nu=0}^{N}L_{1n\nu}^{1+}\left(-1\right)^{\nu}+B_{1}^{*}\right]+\frac{\nu_{01}}{R_{0}}\sum_{\nu=0}^{N}L_{1n\nu}^{3+}\left(-1\right)^{\nu}\right\}$$

$$a_{58}^{\pm}=-k_{3}J_{n}^{\shortmid}\left(k_{3}\right);a_{62}^{+}=B_{1}\left\{\sum_{\nu=1}^{N}L_{2n\nu}^{1+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{\nu_{01}}{R_{0}}n\sum_{\nu=0}^{N}L_{1n\nu}^{1+}\left(-1\right)^{\nu}+\frac{\nu_{01}}{R_{0}}\sum_{\nu=0}^{N}L_{1n\nu}^{3+}\left(-1\right)^{\nu}\right\}$$

$$a_{63}^{+}=B_{1}\left\{\sum_{\nu=1}^{N}L_{3n\nu}^{5+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{\nu_{01}}{R_{0}}n\sum_{\nu=0}^{N}L_{3n\nu}^{1+}\left(-1\right)^{\nu}+\frac{\nu_{01}}{R_{0}}\left[\sum_{\nu=0}^{N}L_{1n\nu}^{1+}\left(-1\right)^{\nu}+B_{1}^{*}\right]\right\}$$

$$a_{64}^{+}=B_{1}\left\{\sum_{\nu=1}^{N}L_{4n\nu}^{5+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{\nu_{01}}{R_{0}}n\sum_{\nu=0}^{N}L_{4n\nu}^{1+}\left(-1\right)^{\nu}+\frac{\nu_{01}}{R_{0}}\sum_{\nu=0}^{N}L_{4n\nu}^{3+}\left(-1\right)^{\nu}\right\}$$

$$a_{65}^{+}=B_{1}\left\{\sum_{\nu=1}^{N}\left[L_{5n\nu}^{5+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}\right]+\frac{\nu_{01}}{R_{0}}n\sum_{\nu=0}^{N}L_{5n\nu}^{1+}\left(-1\right)^{\nu}+\frac{\nu_{01}}{R_{0}}\sum_{\nu=0}^{N}L_{5n\nu}^{3+}\left(-1\right)^{\nu}\right\}$$

$$a_{66}^{\pm}=-\Lambda^{\shortmid}S\gamma_{1}J_{n}^{\shortmid}\left(\gamma_{1}\right);a_{67}^{\pm}=-\Lambda^{\shortmid}S\gamma_{2}J_{n}^{\shortmid}\left(\gamma_{2}\right);a_{68}^{\pm}=-nJ_{n}\left(k_{3}\right);a_{74}^{+}=a_{75}^{+}=\Lambda^{\shortmid};$$

$$a_{79}^{\pm}=-\frac{E_{2}h_{2}}{\left(1-\nu_{02}^{2}\right)R_{0}}\left[k_{1}^{2}J_{n}^{\shortmid\shortmid}\left(k_{1}\right)+\nu_{02}k_{1}J_{n}^{\shortmid}\left(k_{1}\right)-\nu_{02}n^{2}J_{n}\left(k_{1}\right)\right];$$

$$a_{710}^{\pm}=-\frac{E_{2}h_{2}}{\left(1-\nu_{02}^{2}\right)R_{0}}\left[nk_{2}J_{n}^{\shortmid}\left(k_{2}\right)+\nu_{02}nJ_{n}\left(k_{2}\right)-\nu_{02}nJ_{n}^{\shortmid}\left(k_{2}\right)\right];$$

$$a_{81}^{+}=B_{1}\frac{1-\nu_{01}}{2};a_{85}^{+}=B_{1}\frac{1-\nu_{01}}{2}\frac{n}{R_{0}};$$

$$a_{89}^{\pm}=-\frac{E_2h_2}{\left(1-\nu_{02}^2\right)R_0}\left[-nk_1J_n^{\prime}\left(k_1\right)-nk_1J_n^{\prime}\left(k_1\right)+nJ_n\left(k_1\right)\right];$$

$$a_{810}^{\pm}=-\frac{E_2h_2}{\left(1-\nu_{02}^2\right)R_0}\left[-n^2J_n\left(k_2\right)-k_2^2J_n^{\prime\prime}\left(k_2\right)+k_2J_n^{\prime}\left(k_1\right)\right];$$

$$a_{92}^{+}=D_1\left\{\sum_{\nu=1}^{N}L_{2n\nu}^{4+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{n\nu_{01}}{R_0}\left[\sum_{\nu=0}^{N}L_{4n\nu}^{2+}\left(-1\right)^{\nu}+B_1^*\right]\right\};$$

$$a_{91}^{+}=D_1\left\{\sum_{\nu=1}^{N}L_{1n\nu}^{4+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{n\nu_{01}}{R_0}\sum_{\nu=0}^{N}L_{1n\nu}^{2+}\left(-1\right)^{\nu}\right\};$$

$$a_{93}^{+}=D_1\left\{\sum_{\nu=1}^{N}L_{3n\nu}^{4+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{n\nu_{01}}{R_0}\sum_{\nu=0}^{N}L_{3n\nu}^{2+}\left(-1\right)^{\nu}\right\};$$

$$a_{94}^{+}=D_1\left\{\sum_{\nu=1}^{N}\left[L_{4n\nu}^{4+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}\right]+\frac{\nu_{01}}{R_0}n\sum_{\nu=0}^{N}L_{4n\nu}^{2+}\left(-1\right)^{\nu}\right\};$$

$$a_{95}^{+}=D_1\left\{\sum_{\nu=1}^{N}L_{5n\nu}^{4+}\left(-1\right)^{\nu}\beta_{\nu}+\frac{n\nu_{01}}{R_0}\sum_{\nu=0}^{N}L_{5n\nu}^{2+}\left(-1\right)^{\nu}\right\}$$

$$a_{96}^{\pm}=D_1\left[\left(\gamma_1^2S-1\right)J_n^{\prime\prime}\left(\gamma_1\right)+\nu_{02}\left(S\gamma_1-\frac{1}{\gamma_1}\right)J_n^{\prime}\left(\gamma_1\right)+\nu_{02}n^2\left(\frac{1-\gamma_1^2}{\gamma_1^2}\right)\right];$$

$$a_{97}^{\pm}=D_1\left[\left(\gamma_1^2S-1\right)J_n^{\prime\prime}\left(\gamma_2\right)+\nu_{02}\left(S\gamma_2-\frac{1}{\gamma_2}\right)J_n^{\prime}\left(\gamma_2\right)+\nu_{02}n^2\left(\frac{1-\gamma_2^2}{\gamma_2^2}\right)\right];$$

$$a_{98}^{+}=-D_2\left(\frac{1-\nu_{02}}{2}\right)k_3J_n\left(k_3\right);a_{101}^{+}=D_2\left(\frac{1-\nu_{02}}{2R_0}\right);a_{102}^{+}=D_1\left(\frac{1-\nu_{02}}{2}\right);$$

$$a_{104}^{+}=-D_1\left(\frac{1-\nu_{02}}{2}\right)\frac{n}{R_0};a_{106}^{+}=-D_2n\left(1-\nu_{02}\right)\left(S\gamma_1-\frac{1}{\gamma_1}\right)J_n^{\prime}\left(\gamma_1\right);$$

$$a_{107}^{+}=-D_2n\left(1-\nu_{02}\right)\left(S\gamma_2-\frac{1}{\gamma_2}\right)J_n^{\prime}\left(\gamma_2\right);a_{13}^{-}=1;$$

$$a_{98}^{+}=D_2\left(\frac{1-\nu_{02}}{2}\right)\left[2n^2J_n\left(k_3\right)+2k_3J_n^{\prime}\left(k_3\right)-k_3^2J_n\left(k_3\right)\right];$$

$$a_{21}^{-}=\sum_{\nu=1}^{N}L_{1n\nu}^{5-}\left(-1\right)^{\nu};a_{22}^{-}=\sum_{\nu=1}^{N}L_{2n\nu}^{5-}\left(-1\right)^{\nu};a_{23}^{-}=\sum_{\nu=1}^{N}L_{3n\nu}^{5-}\left(-1\right)^{\nu};a_{24}^{-}=\sum_{\nu=1}^{N}L_{4n\nu}^{5-}\left(-1\right)^{\nu};$$

$$a_{25}^{-}=\sum_{\nu=1}^{N}L_{5n\nu}^{5-}\left(-1\right)^{\nu}+B_1^*;a_{51}^{-}=\sum_{\nu=1}^{N}L_{1n\nu}^{4-}\left(-1\right)^{\nu};a_{52}^{-}=\sum_{\nu=1}^{N}L_{2n\nu}^{4-}\left(-1\right)^{\nu};a_{31}^{-}=a_{42=}^{-}1;$$

$$a_{51}^{-}=\sum_{\nu=1}^{N}L_{1n\nu}^{4-}\left(-1\right)^{\nu};a_{52}^{-}=\sum_{\nu=1}^{N}L_{2n\nu}^{4-}\left(-1\right)^{\nu};a_{53}^{-}=\sum_{\nu=1}^{N}L_{3n\nu}^{4-}\left(-1\right)^{\nu};a_{54}^{-}=\sum_{\nu=1}^{N}L_{4n\nu}^{4-}\left(-1\right)^{\nu}+B_1^*;$$

$$a_{55}^{-}=\sum_{\nu=1}^{N}L_{5n\nu}^{4-}\left(-1\right)^{\nu};a_{61}^{-}=B_{1}\frac{\nu_{01}n}{R_{0}};a_{63}^{-}=B_{1}\frac{\nu_{01}}{R_{0}};a_{65}^{-}=B_{1};$$

$$a_{71}^{-}=\Lambda_{1}\left\{\sum_{\nu=1}^{N}L_{1n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\sum_{\nu=0}^{N}L_{1n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$a_{72}^{-}=\Lambda_{1}\left\{\sum_{\nu=1}^{N}L_{2n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{n\nu_{01}}{R_{0}}\sum_{\nu=0}^{N}L_{2n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$a_{73}^{-}=\Lambda_{1}\left\{\sum_{\nu=1}^{N}L_{3n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}+\sum_{\nu=0}^{N}L_{3n\nu}^{4-}\left(-1\right)^{\nu}\right\}$$

$$a_{74}^{-}=\Lambda_{1}\left\{\sum_{\nu=1}^{N}L_{4n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\left[\sum_{\nu=0}^{N}L_{3n\nu}^{4-}\left(-1\right)^{\nu}+B_{1}^{*}\right]\right\}$$

$$a_{75}^{-}=\Lambda_{1}\left\{\sum_{\nu=1}^{N}L_{5n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\sum_{\nu=0}^{N}L_{5n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$a_{76}^{-}=-\Lambda_{1}\left\{\sum_{\nu=1}^{N}\left[L_{5n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+L_{5n\nu}^{4-}\left(-1\right)^{\nu}\right]g_{rn\nu}^{-}\right\}$$

$$a_{81}^{-}=B_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_{0}}\sum_{\nu=1}^{N}L_{1n\nu}^{5-}\left(-1\right)^{\nu}+\left[\sum_{\nu=0}^{N}L_{1n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}\right]\right\};$$

$$a_{82}^{-}=B_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_{0}}\sum_{\nu=1}^{N}L_{2n\nu}^{5-}\left(-1\right)^{\nu}+\sum_{\nu=0}^{N}L_{2n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\};$$

$$a_{83}^{-}=B_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_{0}}\sum_{\nu=1}^{N}L_{3n\nu}^{5-}\left(-1\right)^{\nu}+\sum_{\nu=0}^{N}L_{3n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\};$$

$$a_{84}^{-}=B_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_{0}}\sum_{\nu=1}^{N}L_{4n\nu}^{5-}\left(-1\right)^{\nu}+\sum_{\nu=1}^{N}L_{4n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\};$$

$$a_{85}^{-}=B_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_{0}}\left[\sum_{\nu=1}^{N}L_{5n\nu}^{5-}\left(-1\right)^{\nu}+B_{1}^{*}\right]+\sum_{\nu=0}^{N}L_{5n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\}$$

$$a_{101}^{-}=D_{1}\frac{1-\nu_{02}}{2}\left\{-\frac{1}{R_{0}}\left[\sum_{\nu=1}^{N}L_{1n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}\right]+\sum_{\nu=0}^{N}L_{1n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}-\frac{n}{R_{0}}\sum_{\nu=1}^{N}L_{4n\nu}^{4-}\left(-1\right)^{\nu}\right\}$$

$$a_{102}^{-}=D_{1}\frac{1-\nu_{02}}{2}\left\{\begin{array}{c}\left[\sum_{\nu=1}^{N}L_{2n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}-\frac{2N}{l}\right]-\\-\frac{n}{R_{0}}\sum_{\nu=0}^{N}L_{4n\nu}^{4-}\left(-1\right)^{\nu}-\frac{1}{R_{0}}\sum_{\nu=1}^{N}L_{4n\nu}^{4-}\left(-1\right)^{\nu}\beta_{\nu};\end{array}\right\}$$

$$a_{92}^{-}=D_{1}\frac{\nu_{01}}{R_{0}}n;a_{94}^{-}=D_{1};$$

$$a_{103}^{-}=D_1\frac{1-\nu_{02}}{2}\left\{\sum_{\nu=1}^{N}L_{3n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{1}{R_0}\sum_{\nu=0}^{N}L_{3n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}-\frac{n}{R_0}\sum_{\nu=1}^{N}L_{3n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$a_{104}^{-}=D_1\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_0}\left[\sum_{\nu=1}^{N}L_{4n\nu}^{4-}\left(-1\right)^{\nu}+B_1^{*}\right]+\sum_{\nu=1}^{N}L_{4n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}-\frac{1}{R_0}\sum_{\nu=1}^{N}L_{4n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu};\right\}$$

$$a_{105}^{-}=D_1\frac{1-\nu_{02}}{2}\left\{\sum_{\nu=1}^{N}L_{5n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{1}{R_0}\sum_{\nu=0}^{N}L_{5n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}-\frac{n}{R_0}\sum_{\nu=1}^{N}L_{5n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$F_i^{*\pm}=\sum_{\nu=0}^{N}\overline{C}_{in\nu}^{\pm}f_{rn\nu}^{\pm}+\sum_{\nu=0}^{N}\overline{C}_{in\nu}^{\pm}g_{znj}^{\pm};i=1,2,3.....10$$

$$\overline{C}_{1n\nu}^{+}=-L_{6n\nu}^{3+}\left(-1\right)^{\nu};\overline{C}_{2nj}^{\pm}=A_{3nj}^{*};\overline{C}_{3n\nu}^{+}=-L_{6n\nu}^{1+}\left(-1\right)^{\nu};\overline{C}_{4n\nu}^{+}=-L_{6n\nu}^{2+}\left(-1\right)^{\nu};$$

$$A_{3nj}^{*}=\frac{A_{3nj}-\frac{S}{D_2}}{-\gamma_{jn}^{2}}J_n\left(\gamma_{jn}\right);\overline{C}_{6n\nu}^{+}=-B_1\left[L_{6n\nu}^{5+}+\frac{\nu_{01}n}{R_0}L_{6n\nu}^{1+}+\frac{\nu_{01}}{R_0}L_{6n\nu}^{3+}\right];$$

$$\overline{C}_{9n\nu}^{+}=-D_1\left[L_{6n\nu}^{4+}+\frac{\nu_{01}n}{R_0}L_{6n\nu}^{2+}\right];\overline{C}_{2n\nu}^{-}=-L_{6n\nu}^{5-}\left(-1\right)^{\nu};\overline{C}_{5n\nu}^{-}=-L_{6n\nu}^{4-}\left(-1\right)^{\nu}$$

$$\overline{C}_{9n\nu}^{\prime+}=D_2\left[\frac{A_{3nj}-\gamma_{jn}^{2}SA_{3nj}-\frac{S}{D_2}}{\gamma_{jn}^{2}}\left(\gamma_{jn}^{2}-n^{2}+1\right)\right]J_n\left(\gamma_{jn}\right);$$

$$\overline{C}_{7n\nu}^{-}=-\Lambda_1\left\{L_{6n\nu}^{3-}\left(-1\right)^{\nu}\beta_{\nu}+\sum_{\nu=0}^{N}L_{6n\nu}^{4-}\left(-1\right)^{\nu}\right\};$$

$$\overline{C}_{8n\nu}^{-}=-B_1\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_0}L_{6n\nu}^{5-}\left(-1\right)^{\nu}+L_{6n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\};$$

$$\overline{C}_{10n\nu}^{-}=-D_1\frac{1-\nu_{02}}{2}\left\{-\frac{n}{R_0}L_{6n\nu}^{4-}\left(-1\right)^{\nu}+L_{6n\nu}^{2-}\left(-1\right)^{\nu}\beta_{\nu}+\frac{1}{R_0}L_{6n\nu}^{1-}\left(-1\right)^{\nu}\beta_{\nu}\right\};$$

Solution (49) will be

$$X_i=\sum_{\nu=0}^{N}Z_{n\nu j}^{\pm}f_{rn\nu}^{\pm}+\sum_{j=0}^{N}Z_{n\nu j}^{\prime\pm}g_{rnj}^{\pm};(i=1,2,3....10)\tag{50}$$

where

$$Z_{n\nu j}^{\pm}=\sum_{k=1}^{10}\frac{\overline{C}_{kn\nu}^{\pm}\Delta_{9ki}^{\pm}}{\Delta_{10}^{\pm}};Z_{n\nu j}^{\prime\pm}=\sum_{k=1}^{10}\frac{\overline{C}_{kn\nu}^{\prime\pm}\Delta_{9ki}^{\pm}}{\Delta_{10}^{\pm}}.$$

For $U_{3n\nu}$ and $U_{zn\mu}$ we obtain

$$u_{3n\mu}^{\pm}=L_{1n\mu}^{3\pm}X_1^{\pm}+L_{2n\mu}^{3\pm}X_2^{\pm}+L_{3n\mu}^{3\pm}X_3^{\pm}+L_{4n\mu}^{3\pm}X_4^{\pm}+L_{5n\mu}^{3\pm}X_5^{\pm}+L_{6n\mu}^{3\pm}f_{rn\mu}^{\pm};\tag{51}$$

$$W_{zn\mu}^{\pm}=L_{7n\mu}^{\pm}X_6^{\pm}+L_{8n\mu}^{\pm}X_7^{\pm}+L_{9n\mu}^{\pm}g_{zn\mu}^{\pm};$$

To improve the convergence of the obtained solution we will single-out the characteristic features in unknown functions. It is known that [13, 14] approaching to angle circles on surface $r=1$ and $z=l$ expression for the pressure $P^{e\pm}$ and its derivatives must have the following singularities

a) for the pressure

$$P_n^{e+}(z_0)=A_n\sqrt[3]{\left(l^2-z_0^2\right)^2};P_n^{e-}(z_0)=B_nz_0\sqrt[3]{\left(l^2-z_0^2\right)};P_n^{e\pm}(r_0)=D_n^{\pm}\sqrt[3]{\left(1-r_0\right)^2}; \tag{52}$$

b) for the derivatives from pressure

$$\frac{\partial P_n^{e+}(z_0r)}{\partial r}\mid_{r=1}=\frac{A_n^*}{\sqrt[3]{l^2-z_0^2}};\frac{\partial P_n^{e-}(z_0r)}{\partial r}\mid_{r=1}=\frac{B_n^*z_0}{\sqrt[3]{l^2-z_0^2}};$$

$$\frac{\partial P_n^{e-}(z_0r)}{\partial z}\mid_{z=l}=\frac{D_n^{*\pm}r^n}{\sqrt[3]{1-r_0^2}}. \tag{53}$$

Expanding (52), (53) into Fourier's, Fourier-Bessel's series corresponding to (15) we obtain [12, 15]

$$A_n\sqrt[0]{\left(l^2-z_0^2\right)^2}=\sum_{\nu=0}^{\infty}f_{n\nu}^{s+}\cos\beta_\nu z_0;B_nz_0\sqrt[3]{\left(l^2-z_0^2\right)}=\sum_{\nu=0}^{\infty}f_{n\nu}^{s-}\sin\beta_\nu z_0;$$

$$D_n^{\pm}\sqrt[3]{\left(1-r_0^2\right)^2}=\sum_{\nu=0}^{\infty}g_{nj}^{s-}J_n\left(\gamma_{jn}r\right);\frac{A_n^*}{\sqrt[3]{l^2-z_0^2}}=\sum_{\nu=0}^{\infty}f_{n\nu}^{*s+}\cos\beta_\nu z_0; \tag{54}$$

$$\frac{B_n^*z_0}{\sqrt[3]{l^2-z_0^2}}=\sum_{\nu=0}^{\infty}f_{n\nu}^{*s-}\sin\beta_\nu z_0;\frac{D_n^{*\pm}r^n}{\sqrt[3]{1-r_0^2}}=\sum_{\nu=0}^{\infty}g_{nj}^{*s\pm}J_n\left(\gamma_{jn}r\right)$$

where

$$f_{n\nu}^{s+}=\frac{A_nl^{1/6}\Gamma(5/3)\Gamma(1/2)\Gamma_{7/6}(\beta_\nu l)2^{7/6}}{\beta_\nu^{7/6}};g_{nj}^{*s\pm}=\frac{D_n^{*\pm}\varepsilon_{nj}\Gamma(2/3)\Gamma_{n+7/6}(\gamma_{jn})}{\sqrt[3]{2\gamma_{jn}^2}};$$

$$g_{nj}^{s\pm}=\frac{D_n^{\pm}\varepsilon_{nj}2^{2/3}\Gamma(5/3)\Gamma_{n+5/3}(\gamma_{jn})}{\gamma_{jn}^{5/3}};f_{n\nu}^{*s+}=\frac{A_n\Gamma(2/3)\Gamma(1/2)J_{1/6}(\beta_\nu l)2^{1/6}}{l^{5/6}\beta_\nu^{7/6}};$$

$$f_{n\nu}^{s-}=-\frac{A_n\Gamma(5/3)\Gamma(1/2)J_{13/6}(\beta_\nu l)2^{7/6}l^{7/6}}{l^{5/6}\beta_\nu^{7/6}};f_{n\nu}^{*s-}=-\frac{A_n\Gamma(2/3)\Gamma(1/2)J_{1/6}(\beta_\nu l)2^{1/6}\beta_\nu l}{\beta_\nu^{7/6}}.$$

Unknown coefficients $f_{n\nu}^{e\pm}$, $f_{n\nu}^{*e\pm}$, $g_{nj}^{e\pm}$, $g_{nj}^{*e\pm}$ at large $\nu,j\,(\nu,j>N)$ behave itself like coefficients of known expansions (54), i.e. we could write

$$f_{n\nu}^{e\pm}=\frac{f_{nN}^{e\pm}}{f_{nN}^{s\pm}}f_{n\nu}^{s\pm};f_{n\nu}^{*e\pm}=\frac{f_{nN}^{*e\pm}}{f_{nN}^{*\pm}}f_{n\nu}^{*s\pm};g_{n\nu}^{e\pm}=\frac{g_{nN}^{e\pm}}{g_{nN}^{s\pm}}g_{n\nu}^{s\pm};g_{n\nu}^{*e\pm}=\frac{g_{nN}^{*e\pm}}{g_{nN}^{*\pm}}g_{n\nu}^{*s\pm}. \tag{55}$$

Thus, if in the unlimited systems of algebraic linear equations (24) unknown coefficients are replaced with formulas (55) than elements of N-th columns in matrixes will be:

$$F_{n\mu N}^{*1+}=\frac{(-1)^{\mu}}{f_{nN}^{*s+}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\varkappa\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}f_{n\nu}^{*s+}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{1+}=\frac{(-1)^{\mu}}{f_{nN}^{s+}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\varkappa\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}f_{n\nu}^{s+}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda;$$

$$G_{n\mu N}^{*1+}=\frac{(-1)^{\mu}}{g_{nN}^{*s+}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\varkappa\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{*s+}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$G_{n\mu N}^{1+}=\frac{(-1)^{\mu}}{g_{nN}^{s+}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\varkappa\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{s+}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{*2+}=\frac{J_{n}(\gamma_{jn})}{f_{nN}^{*s+}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\gamma_{jn}^{2}-\lambda^{2}}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}f_{n\nu}^{*s+}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{2+}=\frac{J_{n}(\gamma_{jn})}{f_{nN}^{s+}}\int\left\{\frac{\lambda^{3}J_{n}^{|2}(\lambda)}{\gamma_{jn}^{2}-\lambda^{2}}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}f_{n\nu}^{s+}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda; \qquad (56)$$

$$G_{n\mu N}^{*2+}=\frac{J_{n}(\gamma_{jn})}{g_{nN}^{*s+}}\int\left\{\frac{\lambda^{3}J_{n}^{|2}(\lambda)}{\varkappa\left(\gamma_{jn}^{2}-\lambda^{2}\right)}\left(1+e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{*s+}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$G_{n\mu N}^{2+}=-\frac{J_{n}(\gamma_{jn})}{g_{nN}^{s+}}\int\left\{\frac{\lambda^{3}J_{n}^{|2}(\lambda)}{\left(\gamma_{jn}^{2}-\lambda^{2}\right)}\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{s+}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{*1-}=\frac{(-1)^{\mu}\beta_{\mu}}{f_{nN}^{*s-}}\int\left\{\frac{\lambda J_{n}^{2}(\lambda)}{\varkappa\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}\beta_{\nu}f_{n\nu}^{*s-}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{1-}=\frac{(-1)^{\mu}\beta_{\mu}}{f_{nN}^{s-}}\int\left\{\frac{\lambda J_{n}^{|}(\lambda)J_{n}(\lambda)}{\varkappa\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{(-1)^{\nu}\beta_{\nu}f_{n\nu}^{*s-}}{\varkappa^{2}+\beta_{\nu}^{2}}\right\}d\lambda;$$

$$G_{n\mu N}^{*1-}=\frac{(-1)^{\mu}\beta_{\mu}}{g_{nN}^{*s-}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\varkappa\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{*s-}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$G_{n\mu N}^{1-}=-\frac{(-1)^{\mu}\beta_{\mu}}{g_{nN}^{s-}}\int\left\{\frac{\lambda^{2}J_{n}^{|}(\lambda)J_{n}(\lambda)}{\left(\varkappa^{2}+\beta_{\mu}^{2}\right)}\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{s-}J_{n}(\gamma_{jn})}{\gamma_{jn}^{2}-\lambda^{2}}\right\}d\lambda;$$

$$F_{n\mu N}^{*2-}=\frac{J_n\left(\gamma_{jn}\right)}{f_{nN}^{*s-}}\int\left\{\frac{\lambda^2 J_n^{'}\left(\lambda\right)J_n\left(\lambda\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{\left(-1\right)^{\nu}\beta_{\nu}f_{n\nu}^{*s-}}{\varkappa^2+\beta_{\nu}^2}\right\}d\lambda;$$

$$F_{n\mu N}^{2-}=\frac{J_n\left(\gamma_{jn}\right)}{f_{nN}^{s-}}\int\left\{\frac{\lambda^3 J_n^{'2}\left(\lambda\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)}\left(1-e^{-2\varkappa l}\right)\sum_{\nu=N+1}^{\infty}\frac{\left(-1\right)^{\nu}\beta_{\nu}f_{n\nu}^{s-}}{\varkappa^2+\beta_{\nu}^2}\right\}d\lambda;$$

$$G_{n\mu N}^{*2-}=\frac{J_n\left(\gamma_{jn}\right)}{g_{nN}^{*s-}}\int\left\{\frac{\lambda^3 J_n^{'2}\left(\lambda\right)}{\varkappa\left(\gamma_{jn}^2-\lambda^2\right)}\left(1-e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{*s-}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2}\right\}d\lambda;$$

$$G_{n\mu N}^{*2-}=-\frac{J_n\left(\gamma_{jn}\right)}{g_{nN}^{s-}}\int\left\{\frac{\lambda^3 J_n^{'2}\left(\lambda\right)}{\left(\gamma_{jn}^2-\lambda^2\right)}\left(1+e^{-2\varkappa l}\right)\sum_{j=N+1}^{\infty}\frac{g_{nj}^{s-}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2}\right\}d\lambda.$$

where

$$\frac{2^{1/6}l^{1/6}\Gamma\left(2/3\right)\Gamma\left(1/2\right)I_{1/6}\left(\varkappa l\right)}{\varkappa^{7/6}sh\left(\varkappa l\right)}-\sum_{\nu=0}^{N-1}\frac{f_{n\nu}^{*s+}\left(-1\right)^{\nu}}{\varkappa^2+\beta_{\nu}^2}=\sum_{\nu=N+1}^{\infty}\frac{f_{n\nu}^{*s+}\left(-1\right)^{\nu}}{\varkappa^2+\beta_{\nu}^2};$$

$$\frac{2^{7/6}l^{7/6}\Gamma\left(5/3\right)\Gamma\left(1/2\right)I_{7/6}\left(\varkappa l\right)}{\varkappa^{13/6}sh\left(\varkappa l\right)}-\sum_{\nu=0}^{N-1}\frac{f_{n\nu}^{s+}\left(-1\right)^{\nu}}{\varkappa^2+\beta_{\nu}^2}=\sum_{\nu=N+1}^{\infty}\frac{f_{n\nu}^{s+}\left(-1\right)^{\nu}}{\varkappa^2+\beta_{\nu}^2};$$

$$\frac{2^{2/3}\lambda^{1/3}\Gamma\left(2/3\right)J_{n+5/3}\left(\lambda\right)}{\left(\lambda^2-n^2\right)J_n^2\left(\lambda\right)J_n^{'}\left(\lambda\right)}-\sum_{\nu=0}^{N-1}\frac{g_{n\nu}^{*s\pm}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2}=\sum_{\nu=N+1}^{\infty}\frac{g_{n\nu}^{*s\pm}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2};$$

$$\frac{2^{2/3}\lambda^{-2/3}\Gamma\left(5/3\right)\Gamma\left(1/2\right)J_{n+2/3}\left(\lambda\right)}{\left(\lambda^2-n^2\right)J_n^2\left(\lambda\right)J_n^{'}\left(\lambda\right)}-\sum_{\nu=0}^{N-1}\frac{g_{n\nu}^{s\pm}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2}=\sum_{\nu=N+1}^{\infty}\frac{g_{n\nu}^{s\pm}J_n\left(\gamma_{jn}\right)}{\gamma_{jn}^2-\lambda^2};$$

$$\frac{2^{1/3}\Gamma\left(2/3\right)\Gamma\left(1/2\right)\left[\varkappa l I_{1/6}^{'}\left(\varkappa l\right)-1/6I_{1/6}\left(\varkappa l\right)\right]}{\varkappa^{1/6}sh\left(\varkappa l\right)}-\sum_{\nu=0}^{N-1}\frac{f_{n\nu}^{*s-}\left(-1\right)^{\nu}\beta_{\nu}}{\varkappa^2+\beta_{\nu}^2}=\sum_{\nu=N+1}^{\infty}\frac{f_{n\nu}^{*s-}\left(-1\right)^{\nu}\beta_{\nu}}{\varkappa^2+\beta_{\nu}^2}$$

$$\frac{2^{7/6}l^{13/6}\Gamma\left(5/3\right)\Gamma\left(1/2\right)\left[\varkappa l I_{7/6}^{'}\left(\varkappa l\right)-7/6I_{7/6}\left(\varkappa l\right)\right]}{\varkappa^{13/6}sh\left(\varkappa l\right)}-\sum_{\nu=0}^{N-1}\frac{f_{n\nu}^{s+}\left(-1\right)^{\nu}\beta_{\nu}}{\varkappa^2+\beta_{\nu}^2}=\sum_{\nu=N+1}^{\infty}\frac{f_{n\nu}^{s+}\left(-1\right)^{\nu}\beta_{\nu}}{\varkappa^2+\beta_{\nu}^2}$$

Substituting the obtained values into condition of hydro-resilient contact (9) and taking into account (24), (34), (35), (56) we obtain such unlimited systems written in matrix form to evaluate the unknown coefficients

$$f_{n\nu}^{e\pm},f_{n\nu}^{*e\pm},g_{nj}^{e\pm},g_{nj}^{*e\pm},f_{n\nu}^{o\pm},f_{n\nu}^{*o\pm},g_{nj}^{o\pm},g_{nj}^{*o\pm};\left(\nu,j=0,1,2.....N\right);$$

$$\underset{N\times N}{\left(F_{1n}^{1\pm}\right)_{\nu\mu}}\underset{1\times N}{\left\{f_n^{e\pm}\right\}_{\nu}}+\underset{N\times N}{\left(F_{2n}^{*1\pm}\right)_{\nu\mu}}\underset{1\times N}{\left\{f_n^{*e\pm}\right\}_{\nu}}+\underset{N\times N}{\left(G_{1n}^{1\pm}\right)_{\nu\mu}}\underset{1\times N}{\left\{g_n^{e\pm}\right\}_{j}}+\underset{N\times N}{\left(G_{2n}^{*1\pm}\right)_{\nu\mu}}\underset{1\times N}{\left\{g_n^{*e\pm}\right\}_{j}}=0; \tag{57}$$

$$\underset{N\times N}{\left(F_{1n}^{2\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{e\pm}\right\}}_{\nu}+\underset{N\times N}{\left(F_{2n}^{*2\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{*e\pm}\right\}}_{\nu}+\underset{N\times N}{\left(G_{1n}^{2\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_n^{e\pm}\right\}}_{j}+\underset{N\times N}{\left(G_{2n}^{*2\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_n^{*e\pm}\right\}}_{j}=0 \tag{58}$$

$$\underset{N\times N}{\left(R_{1n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_{rn}^{\pm}\right\}}_{\nu}+\underset{N\times N}{\left(R_{2n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_{zn}^{\pm}\right\}}_{j}=\underset{1\times N}{\left\{f_n^{*e\pm}\right\}}_{\nu}+\underset{1\times N}{\left\{f_n^{*a\pm}\right\}}_{\nu}; \tag{59}$$

$$\underset{N\times N}{\left(R_{3n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_{rn}^{\pm}\right\}}_{\nu}+\underset{N\times N}{\left(R_{4n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_{zn}^{\pm}\right\}}_{j}=\underset{1\times N}{\left\{g_n^{*e\pm}\right\}}_{j}+\underset{1\times N}{\left\{g_n^{*a\pm}\right\}}_{j}; \tag{60}$$

$$\underset{N\times N}{\left(R_{1n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_{rn}^{\pm}\right\}}_{\nu}+\underset{N\times N}{\left(R_{2n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_{zn}^{\pm}\right\}}_{j}=\underset{N\times N}{\left(E_{1n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{*o\pm}\right\}}_{\nu}; \tag{61}$$

$$\underset{N\times N}{\left(R_{3n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_{rn}^{\pm}\right\}}_{\nu}+\underset{N\times N}{\left(R_{4n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{g_{zn}^{\pm}\right\}}_{j}=\underset{N\times N}{\left(E_{2n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{*o\pm}\right\}}_{\nu} \tag{62}$$

$$\underset{N\times N}{\left(E_{3n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{*o\pm}\right\}}_{\nu}+\underset{N\times N}{\left(M_{1n}^{\pm}\right)}_{j\mu}\underset{1\times N}{\left\{g_n^{*o\pm}\right\}}_{j}=\underset{1\times N}{\left\{f_n^{o\pm}\right\}}_{\nu}; \tag{63}$$

$$\underset{N\times N}{\left(E_{4n}^{\pm}\right)}_{\nu\mu}\underset{1\times N}{\left\{f_n^{*o\pm}\right\}}_{\nu}+\underset{N\times N}{\left(M_{2n}^{\pm}\right)}_{j\mu}\underset{1\times N}{\left\{g_n^{*o\pm}\right\}}_{j}=\underset{1\times N}{\left\{g_n^{o\pm}\right\}}_{\nu} \tag{64}$$

where

$$\overline{R}_{in\nu\mu}=\rho\omega^2c^2R_{in\nu\mu};i=1,2,3,4.$$

$$f_{rn\nu}^{\pm}=f_{n\nu}^{e\pm}+f_{n\nu}^{a\pm}-f_{n\nu}^{o\pm};g_{znj}^{\pm}=g_{nj}^{e\pm}+g_{nj}^{a\pm}-g_{nj}^{o\pm};$$

$$E_{4n\mu\nu}^{-}=\left[\sum_{\nu=1}^{N}C_{n\nu\mu}^{-}J_n\left(\gamma_{jn}\right)\left(-1\right)^{\nu}\beta_{\nu}+\frac{2N}{l}\right]\delta_{\nu\mu};$$

$$E_{3n\mu\nu}^{\pm}=J_n\left(\varkappa_2\right)\delta_{\nu\mu};E_{2n\mu\nu}^{\pm}=\frac{\beta_0}{\xi_{02}\varepsilon_{n\mu}}\delta_{\nu\mu};E_{1n\mu\nu}^{\pm}=\frac{\varkappa_2J_n^{\shortmid}\left(\varkappa_2\right)}{\xi_{01}\beta_0^{-1}};$$

$$E_{4n\mu\nu}^{+}=\left\{\left[\sum_{\nu=1}^{N}C_{n\nu\mu}^{+}\varepsilon_{\mu n}\left(-1\right)^{\nu}\right]+\frac{2}{\pi l}\sum_{\nu=N+1}^{\infty}\frac{1}{\nu^2}\right\}\delta_{\nu\mu}$$

$$M_{1n\mu\nu}^{\pm}=C_{n\nu\mu}^{\pm}J_n\left(\gamma_{jn}\right);M_{2n\mu\nu}^{+}=\frac{\left(-1\right)^{\nu}\varkappa_2J_n^{\shortmid}\left(\varkappa_2\right)J_n\left(\gamma_{jn}\right)\varepsilon_{\mu n}}{\gamma_{\mu n}^2-\varkappa_2^2};$$

$$M_{2n\mu\nu}^{+}=\frac{\left(-1\right)^{\nu}\varkappa_2J_n^{\shortmid}\left(\varkappa_2\right)J_n\left(\gamma_{jn}\right)\varepsilon_{\mu n}\beta_{\nu}}{\gamma_{\mu n}^2-\varkappa_2^2}.$$

Thus we obtain the closed finite system of $8\left(2N+1\right)$ linear algebraic equations for the symmetrical and anti-symmetrical components of the solution of given problem. From the system (57) -(64) we define the unknown coefficients

$$f_{n\nu}^{e\pm},f_{n\nu}^{*e\pm},g_{nj}^{e\pm},g_{nj}^{*e\pm},f_{n\nu}^{o\pm},f_{n\nu}^{*o\pm},g_{nj}^{o\pm},g_{nj}^{*o\pm};\nu,j=0,1,2....N$$

Let us consider some particular cases, which are directly obtained from (57):

1) If liquid filler is absent inside the cylindrical cover than we obtain the soluted system of first four matrix equations (57) -(60) with the following modifications

$$f_{n\nu}^{o\pm}=0;g_{nj}^{o\pm}=0;$$

2) For absolute solid (soft) cylinder the soluted system will be formed of matrix equations (57), (58) and such two equations

$$\underset{N\times 1}{\left\{f_n^{*e\pm}\right\}_\nu}=\underset{N\times 1}{\left\{f_n^{*a\pm}\right\}_\nu};\underset{N\times 1}{\left\{g_n^{*e\pm}\right\}_j}=\underset{N\times 1}{\left\{g_n^{*a\pm}\right\}_j};\underset{N\times 1}{\left\{f_n^{e\pm}\right\}_\nu}=\underset{N\times 1}{\left\{f_n^{a\pm}\right\}_\nu};\underset{N\times 1}{\left\{g_n^{e\pm}\right\}_j}=\underset{N\times 1}{\left\{g_n^{a\pm}\right\}_j}; \tag{65}$$

3) If the cylindrical cover is replaced with liquid cylinder we obtain the soluted system containing (57), (58), (63), (64) matrix equations and such matrix equations:

$$\underset{N\times 1}{\left\{f_n^{*e\pm}\right\}_\mu}+\underset{N\times 1}{\left\{f_n^{*a\pm}\right\}_\mu}=\underset{N\times N}{\left(E_{1N}^{\pm}\right)_{\nu\mu}}\underset{N\times 1}{\left\{f_n^{*o\pm}\right\}_{\mu\nu}};\underset{N\times 1}{\left\{g_n^{*e\pm}\right\}_\mu}+\underset{N\times 1}{\left\{g_n^{*a\pm}\right\}_\mu}=\underset{N\times N}{\left(E_{2N}^{\pm}\right)_{j\mu}}\underset{N\times 1}{\left\{g_n^{*o\pm}\right\}_\mu};$$

$$\underset{N\times 1}{\left\{f_n^{e\pm}\right\}_\mu}+\underset{N\times 1}{\left\{f_n^{a\pm}\right\}_\mu}=\underset{N\times 1}{\left\{f_n^{o\pm}\right\}_\mu};\underset{N\times 1}{\left\{g_n^{e\pm}\right\}_\mu}+\underset{N\times 1}{\left\{g_n^{a\pm}\right\}_\mu}=\underset{N\times 1}{\left\{g_n^{o\pm}\right\}_\mu}; \tag{66}$$

4) If point pressure source

$$P^o=P_0\delta\left(r-r_0\right)\delta\left(\theta\right)/4\pi r;$$

is placed inside the cylindrical cover than the system of matrix equations (57) - (64) must be changed as follows

$$f_{n\nu}^{*a+}=f_{n\nu}^{*a-}=g_{nj}^{a-}=g_{nj}^{*a+}=0; \tag{67}$$

$$g_{nj}^{a+}=-\sum_{\nu=0}^{\infty}\frac{P_0\varepsilon_\nu\varepsilon_{nj}\left(-1\right)^\nu J_n\left(\gamma_{jn}\right)\cos\beta_\nu z_0}{\pi l};g_{nj}^{*a-}=-\sum_{\nu=0}^{\infty}\frac{P_0\beta_\nu\varepsilon_{nj}\left(-1\right)^\nu J_n\left(\gamma_{jn}\right)\sin\beta_\nu z_0}{\pi l};$$

$$f_{nj}^{a-}=-\sum_{j=0}^{\infty}\frac{P_0\varepsilon_{nj}J_n^2\left(\gamma_{jn}\right)\sin\beta_\nu z_0}{\pi l};f_{nj}^{a+}=-\sum_{\nu=0}^{\infty}\frac{P_0\varepsilon_\nu\varepsilon_{nj}J_n\left(\gamma_{jn}\right)\cos\beta_\nu z_0}{\pi l}.$$

Thus, in this work a new problem solution methodology of contact interaction of acoustic medium with resilient finite bodies of cylindrical form, based on application of boundary integral equations method in conjunction with series method with later use of series convergence improvement taking into account the particularities of the determined functions.

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