Defeating classical bit commitments with a quantum computer

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Abstract

It has been recently shown by Mayers that no bit commitment scheme is secure if the participants have unlimited computational power and technology. However it was noticed that a secure protocol could be obtained by forcing the cheater to perform a measurement. Similar situations had been encountered previously in the design of Quantum Oblivious Transfer. The question is whether a classical bit commitment could be used for this specific purpose. We demonstrate that, surprisingly, classical unconditionally concealing bit commitments do not help.

1 Introduction

After Mayers obtained his general impossibility theorem for bit commitment schemes (see the Appendix of [13] and [1, 2]), different kind of ideas were proposed by some of us with the hope to realize unconditionally secure bit commitment. It was then realized that these apparently promising ideas were also ruled out by Mayers' attack. These attempts contributed to enhance our understanding of what is going on with quantum bit commitment. However, no complete discussion on the subject has ever been provided in the literature. The most interesting attempts were based on the use of a classical bit commitment together with temporary assumptions on the power of the cheater. The idea was to use the classical bit commitment to force the cheater

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to perform a measurement. This would be useful to realize many quantum protocols other than quantum bit commitments. Here our objective is to bring out the general principles that explain why this approach does not work in quantum cryptography.

Before we proceed, let us briefly explain the notion of bit commitment and its impact in quantum cryptography. Quantum cryptography is often associated with a cryptographic application called key distribution [11, 12] and it has achieved success in this area [3]. However, other applications of quantum mechanics to cryptography have also been considered and bit commitment was at the basis of most if not all of these other applications [3, 14, 15, 16]. A *bit commitment* scheme allows $\mathcal{A}lice$ to send something to $\mathcal{B}ob$ that commits her to a bit b of her choice in such a way that $\mathcal{B}ob$ cannot tell what b is, but such that $\mathcal{A}lice$ can later prove to him what b originally was. You may think of this as $\mathcal{A}lice$ sending a note with the value b written on it in a strong-box to $\mathcal{B}ob$ and later revealing him the combination to the safe.

The commitment obtained after the commit phase is binding if $\mathcal{A}lice$ cannot change the value of b and it is concealing if $\mathcal{B}ob$ cannot obtain any information about b without the help of $\mathcal{A}lice$. The commitment is secure if it is binding and concealing. The commitment is unconditionally secure if it is secure against a cheater, either $\mathcal{A}lice$ or $\mathcal{B}ob$, with unlimited technology and computational power. In 1993 a protocol for quantum bit commitment, henceforth referred to as BCJL, was thought to be *provably secure* [15]. Because of quantum bit commitment, the future of quantum cryptography was very bright, with new applications such as the identification protocol of Crépeau and Salvail [17] coming up regularly.

The trouble began in October 1995 when Mayers found a subtle flaw in the BCJL protocol. Though Mayers explained his discovery to many researchers interested in quantum bit commitment [18], his result was not made entirely public until after Lo and Chau discovered independently a similar result in March 1996 [19]. The result of Mayers was more general than the one obtained by Lo and Chau, but both used the same basic idea. The result of Lo and Chau did not encompass the BCJL protocol in which $\mathcal{B}ob$ can obtain an exponentially small amount of information. (In practice a protocol is considered secure as long as $\mathcal{B}ob$ cannot obtain more than an exponentially small amount of information on the bit committed by $\mathcal{A}lice$, that is, an amount of information that goes exponentially fast to 0 as the number of photons used in the protocol increases.) However, the final version published by Lo and Chau [19] used the techniques previously used by Mayers [18] to prove the non security of the BCJL protocol and any other protocol published at the time. So, the paper of Lo and Chau [19] is a proper account of these preliminary results.

2 The general impossibility theorem

Now, we review the general theorem [2], which says that a quantum protocol that creates an unconditionally secure bit commitment is simply impossible. The main additional difficulty in the general result is that it is easy to think that measurements and classical communication could be used to restrict the behaviour of the cheater during the commit phase, and thus obtain a secure bit commitment. In fact, after BCJL was shown not secure, the spontaneous attitude was to try alternative quantum bit commitment protocols by making some clever use of measurements and classical communication [20]. Some of these protocols were proposed after Mayers obtained the general result in March 1996. All of these protocols were found not secure against Mayers' attack.

There exists two approaches to deal with measurements and classical communication in quantum bit commitment protocols: an indirect and a direct approach. In the first proof of Mayers (see [1] and the Appendix of [13]) the indirect approach was used. It was shown that any protocol in which classical information is used is equivalent to another protocol in which no classical information is used. Then it was shown that no such protocol is unconditionally secure. The advantage of this approach is that, after the reduction is shown, the attack on the new protocol is easy to describe and analyze because there is no classical communication anymore. The disadvantage is that we don't deal directly with the issue of classical communication and measurements, that is, the attack obtained against the new protocol is not the one that applies on the original protocol. The attack on the new protocol does not include any classical communication, whereas in the original protocol the cheater must communicate classically with the honest participant (otherwise this honest participant will wonder what is going on).

We emphasize that the proof of the reduction which is not that hard must nevertheless explain why the cheater can still cheat in the original protocol despite the fact that he is restricted by measurements and decoherence which must occur because of classical communication. Otherwise the overall proof would simply miss the important issue of classical communication – it would not encompass the protocols and ideas that have been proposed recently [3, 5, 8, 9]. Mayers preferred to use a more direct approach without reduction in [2]. So, Mayers' paper [2] directly describes and analyzes the real attack that must be performed by the cheater.

Lo and Chau also wrote a paper [7] to discuss the issue of quantum communication and other aspects of Mayers' result. They used a variant of Yao's model for quantum communication. The essence of this model is that a third system is passed back and forth under the control of each participant when it is their turn [16]. Mayers' attack works fine in this model, and it is indeed important to verify that the attack works in such a reasonable model. With regard to classical communication, the discussion of Lo and Chau [7] is similar to the indirect approach of Mayers.

Now, let us consider the attack. Of course, we are interested in the attack on the original protocol. The attack on the new protocol is just a construction in a proof. We emphasize that in both approaches, with a reduction or without a reduction, the attack on the original protocol is the same. Here we focus on the part of the attack that must be performed during the commit phase. (The remainder of the attack, which is performed after the commit phase, is the same as when there is no classical communication, so it creates no additional difficulty.) One ingredient in the attack is that the cheater keeps everything at the quantum level except what must be announced classically. Assume that at some given stage of the commit phase, a participant has normally generated a classical random variable R, performed measurements to obtain an overall outcome X, and shared some classical information Y with the other participant as a result of previous communication. Now, assume that this participant is the cheater and that the protocol says he must transmit some classical information $f(R, X, Y)$, which for simplicity we assume is a binary string. One might think that the cheater must have generated the random variable R and the outcome X in order to be able to compute and send $f(R, X, Y)$. However, the cheater does not have to do that. As we explain later, he can do the entire computation of $f(R, X, Y)$, including the computation of R and the measurements, at the quantum level. Only Y needs to be classical. Then he can measure the bits of the string $f(X, R, Y)$ (only these bits) and send them to the other participant. The final result is that all information is kept at the quantum level, except what must be sent classically to the other participant. As explained in [1, 2] (see also the Appendix of [13]) this strategy performed during the commit phase either allows $\mathcal{B}ob$ to obtain some information about the bit committed by $\mathcal{A}lice$, without any help from $\mathcal{A}lice$, or else allows $\mathcal{A}lice$ to change her mind after the commit phase.

To understand how the cheater can perform the same algorithm at the quantum level, it is useful to keep in mind that any classical process can be seen as a special kind of quantum phenomena. Therefore, in principle no modification is required because the classical algorithm already describes a quantum process. It is sufficient to describe the exact same algorithm in the viewpoint that every phenomena corresponds to a unitary transformation. The standard way to do that is the following. Let $\mathcal{C}(\omega_0)$ be the initial state of a measuring apparatus. A measurement on a state ψ becomes a unitary transformation that maps $\psi \otimes \mathcal{C}(\omega_0)$ into $\sum_i \psi_i \otimes \mathcal{C}(\omega_i)$ where $\mathcal{C}(\omega_i)$ is the state of the measuring apparatus associated with the outcome ω_i and ψ_i is the corresponding (unnormalized) final state of the measured system. The generation of a random variable r with probability $p(r)$ corresponds to the creation of a superposition $\sum_r \sqrt{p(r)}\mathcal{C}(r)$ where the states $\mathcal{C}(r)$ are orthonormal states that encode the random classical information r. In particular, the state $\psi = \alpha\mathcal{C}(0) + \beta\mathcal{C}(1)$ corresponds to a random bit that takes the values 0 and 1 with probability $|\alpha|^2$ and $|\beta|^2$ respectively. Now, suppose that a function $f(x)$ must be computed. The output of the function f requires a new register denoted F. Initially, the registers X and F are in state $\sum_x \lambda_x \mathcal{C}(x) \otimes \mathcal{C}(0)$, that is, the value x for the input occurs with probability $|\lambda_x|^2$ and the register F is initialized at 0. The result of the computation of f is the state $\sum_x \lambda_x \mathcal{C}(x) \otimes \mathcal{C}(f(x))$. When we say that the cheater performs the same algorithm at the quantum level, we mean that the classical states $\mathcal{C}(x)$, $\mathcal{C}(r)$, etc. are replaced by orthogonal quantum states of truly quantum systems. These techniques will become clear when examples are discussed in the next section.

Despite the fact that formally the algorithms are identical, the cheater will have more flexibility later on if he performs his algorithm at the quantum level rather than at the classical level. It is not true that these two levels are equivalent. For example, the truly quantum state $\alpha|0\rangle + \beta|1\rangle$ can be unitarily mapped into the state $|0\rangle$, but this is not true for the corresponding classical state because a part of the overall states $\mathcal{C}(0)$ and $\mathcal{C}(1)$ is encoded in an irreversible manner in the environment or the classical apparatus, etc. Therefore, one would like to find a way to force the cheater to perform real measurements, as requested in the honest protocol. This would be useful not only to realize quantum bit commitment protocols, but to realize many other quantum protocols, including the important quantum oblivious transfer protocols [23, 14].

A better understanding of the situation came after Crépeau proposed a quantum protocol [3, 5] that uses a computationally secure classical bit commitment [21, 22] as a subprotocol. The idea was to rely temporarily on the limitation (in speed) on the cheater during the commit phase to force him to perform some measurements. The hope was that this short-term assumption could be dropped after the commit phase so as to obtain a quantum bit commitment not relying on any long-term assumption. Salvail also proposed a protocol in which two participants, $\mathcal{A}lice$ and $\mathcal{A}lyson$ say, want to commit a bit to $\mathcal{B}ob$. $\mathcal{A}lice$ and $\mathcal{A}lyson$ are sufficiently far apart that they cannot communicate during the commit phase. Again the hope was that this temporary restriction on the cheaters during the commit phase would be sufficient to obtain a secure quantum bit commitment not relying on any long-term assumption.

However, after some thoughts, one realizes that the cheater in Mayers' attack performs the honest algorithm: the only difference is that he performs this honest algorithm at the quantum level. Therefore, if the cheater has the power to perform the honest protocol (which he must have) and has the technology to store information at the quantum level, then he has the power to cheat during the commit phase, despite the fact that he has not the power to break the computationally secure bit commitment efficiently, or despite the fact that $\mathcal{A}lice$ and $\mathcal{A}lyson$ cannot communicate during the commit phase. After the commit phase, the rule of the game is that we must drop the assumption on the computational power of the cheater, so the fact that a computationally secure bit commitment was used is irrelevant: the proof applies.

3 Quantum attacks

Here, we analyze the possibility to use classical bit commitment protocol to force the cheater to perform a measurement. Our conclusion is that, surprisingly, a whole class of classical BC schemes (that are perfectly concealing) fail miserably in this scenario. Our result is illustrated with the computational BC scheme of Naor, Ostrovsky, Venkatesen and Yung [22], and the two-prover BC scheme of Ben-Or, Goldwasser, Kilian and Wigderson [6]. The basic idea can be used regardless of the BC scheme.

The attack is inspired from the discussion of the previous section, but we will focus on the fact that the objective (defeated by the attack) is to force a measurement rather that an entire protocol. In the following, the goal of the protocol is to force a measurement using a classical bit commitment. In the cheating protocol, $\mathcal{A}lice$ creates a superposition of all possible honest strategies. For example, suppose that $\mathcal{A}lice$ is given a quantum state $\psi = \alpha|0\rangle + \beta|1\rangle$ to measure and that Bob is expecting a commitment to the outcome. In the cheating protocol, Alice measures the state ψ at the quantum level as explained in the previous section. Then she performs the commit part of the (classical) protocol at the quantum level as if she committed to the outcome of the measurement. At the end of the commit phase, this outcome is entangled with other registers on Alice's side, but it is still in superposition. At this point there are two possibilities.

- if unveiling is requested, she measures her remaining quantum state and successfully complete the protocol as if she had been honest all the way!
- if no unveiling is requested, she undoes the entanglement in such a way as to recover the state ψ that was given to her to start with, completely untouched!

This completely defeats the purpose of the BC scheme if the goal was to force a measurement. We now illustrate this principle with two examples.

3.1 NOVY Bit Commitment Scheme

In a computational scenario two techniques reduce bit commitment to very general cryptographic assumptions: the protocol of Naor [10] reducing unconditionally binding and computationally concealing bit commitment to pseudorandomness, and the protocol of Naor, Ostrovsky, Venkatesen and Yung [22] reducing computationally binding and unconditionally concealing bit commitment to one-way permutations. We restrict our attention to this second result and first present their construction. In the following, $\pi : \{0,1\}^n \to \{0,1\}^n$ denotes a one-way permutation.

Protocol 3.1 ($NOVY/Commit(b)$)

1: $\mathcal{A}lice$ picks $x \in_{\mathrm{R}} \{0,1\}^n$, and computes $y := \pi(x)$,

2: **for** $i \in \{1, ..., n-1\}$ **do**

3: $\mathcal{B}ob$ picks a hash vector $h_i \in_{\mathrm{R}} \{0,1\}^n$ and announces it to $\mathcal{A}lice$,

4: $\mathcal{A}lice$ announces $r_i := h_i \cdot y$ to $\mathcal{B}ob$,

5: **endfor**

6: Let y_0, y_1 be two solutions to $\{r_i = h_i \cdot y_*\}_{1 \le i < n}$ in some fixed order (say $y_0 < y_1$). Alice announces $z := a \oplus b$ to $\mathcal{B}ob$ where a is such that $y = y_a$.

The fact that this protocol is unconditionally concealing is obvious since the commitment depends entirely on the fact that $\mathcal{A}lice$ knows the inverse of y_0 or y_1. Since both have a unique inverse, it is impossible for $\mathcal{B}ob$ to tell which one $\mathcal{A}lice$ knows. Intuitively, the reason why this protocol is binding is that the problem of finding two couples (x_0, y_0) and (x_1, y_1) such that $y_0 = \pi(x_0)$, $y_1 = \pi(x_1)$ and $h_i \cdot y_0 = h_i \cdot y_1, 1 \le i < n$ is difficult.

Protocol 3.2 ($NOVY/Unveil(b)$)

1: $\mathcal{A}lice$ discloses b and x to $\mathcal{B}ob$,

2: $\mathcal{B}ob$ checks that $y_{z \oplus b} = \pi(x)$.

Naor, Ostrovsky, Venkatesen and Yung showed that it is computationally equivalent to cheat the unveiling protocol or to inverse the one-way permutation. Any efficient algorithm to solve one problem yields an efficient algorithm to solve the other. Their proof technique involves an algorithm to convert any attacker A to the commitment scheme to an inverter I of the one-way permutation.

3.1.1 NOVY on a quantum computer

In order to describe the cheating protocol, we use standard tricks of quantum computation. For the reader unfamiliar with quantum computing we recommend [4] as an introduction. We now describe precisely the attack.

The attack. In the cheating protocol, the state $\psi = \alpha|0\rangle + \beta|1\rangle$ is the input bit b in superposition. So, we denote B the register that contains the state ψ.

Protocol 3.3 (*NOVY/Commit*$(*)$)

1: $\mathcal{A}lice$ chooses x and computes $y = \pi(x)$ at the quantum level, that is, she sets up quantum registers X, Y in state $\sum_{x \in \{0,1\}^n} \frac{1}{\sqrt{2^n}} |x, \pi(x)\rangle$,

2: **for** $i \in \{1, ..., n-1\}$ **do**

3: $\mathcal{B}ob$ picks a hash vector $h_i \in_{\mathrm{R}} \{0,1\}^n$ and announces it to $\mathcal{A}lice$,

4: Let $S_i = \{x | h_j \cdot \pi(x) = r_j, \text{for} 1 \leq j < i\}$.
$\mathcal{A}lice$ computes $r_i = h_i \cdot y$ at the quantum level, that is, she sets up registers X, Y, R in state $\sum_{x \in S_i} \frac{1}{\sqrt{2^{n-i+1}}} |x, \pi(x), h_i \cdot \pi(x)\rangle$, and announces r_i, the outcome of measuring R, to $\mathcal{B}ob$.

5: **endfor**

{ At this point S_{n-1} contains two solutions x_0, x_1 to $\{r_i = h_i \cdot \pi(x)\}_{1 \leq i < n}$.
So the state of the registers B, X, Y is

$$\begin{aligned} &(\alpha|0\rangle + \beta|1\rangle) \otimes \frac{1}{\sqrt{2}}(|x_0, y_0\rangle + |x_1, y_1\rangle) \\ = \quad & \frac{\alpha}{\sqrt{2}}|0, x_0, y_0\rangle + \frac{\alpha}{\sqrt{2}}|0, x_1, y_1\rangle + \frac{\beta}{\sqrt{2}}|1, x_0, y_0\rangle + \frac{\beta}{\sqrt{2}}|1, x_1, y_1\rangle. \end{aligned}$$

}

6: $\mathcal{A}lice$ computes $z := a \oplus b$, where a is the index of y, at the quantum level, that is, she prepares the registers B, X, Y, Z in the state

$$\frac{\alpha}{\sqrt{2}}|0, x_0, y_0, 0\rangle + \frac{\alpha}{\sqrt{2}}|0, x_1, y_1, 1\rangle + \frac{\beta}{\sqrt{2}}|1, x_0, y_0, 1\rangle + \frac{\beta}{\sqrt{2}}|1, x_1, y_1, 0\rangle.$$

Then she measures the register Z and announces z to $\mathcal{B}ob$.

Protocol 3.4 (*NOVY/Unveil*$(*)$)

1: $\mathcal{A}lice$ measures registers B and X, and announces b and x to $\mathcal{B}ob$,

2: $\mathcal{B}ob$ checks that $y_{z \oplus b} = \pi(x)$.

If $\mathcal{A}lice$ unveils. First we want to verify that, if $\mathcal{A}lice$ unveils the bit, she passes the test. Perhaps the easier way to verify this fact is to actually compute the state she has after the commit part. If she announces $z = 0$, the state of B, X, Y is

$$\alpha|0, x_0, y_0\rangle + \beta|1, x_1, y_1\rangle.$$

If she announces $z = 1$, the state of B, X, Y is

$$\alpha|0, x_1, y_1\rangle + \beta|1, x_0, y_0\rangle.$$

Note that in both cases, the bit b has the correct distribution of probability (the one associated with the initial state ψ). Also, one may easily check that $z \oplus b = a$ so that $y_{z\oplus b} = y = \pi(x)$. Therefore, *Alice* passes the test. There is another way to see the same result. To unveil the bit b, *Alice* measures the registers B and X. Because these registers are only used as control registers, *Alice* could measure them at the beginning of the protocol (just after the first step) and this would make no difference (as far as the distribution of probabilities is concerned). This can actually be verified by checking that the operation associated with a measurement on B and X (in the computational basis) and the operation associated with a computation where B and X are control registers always commute. In the viewpoint where she measures these registers at the beginning of the protocol, we are back to the honest classical protocol because all superpositions disappear. Clearly, if *Alice* is honest she should pass the test.

If *Alice* never Unveils. Second we observe that if $NOVY/Unveil(*)$ does not take place, then *Alice* may recover ψ from her registers B, X, Y. She has only to compute (x_a, y_a) at the quantum level using $a = b \oplus z$ and then erase the registers X and Y using a bitwise XOR, and discard these registers. Note that to compute x_a, she needs to compute $f^{-1}(y_a)$ because she does not know (x_0, x_1), she only knows (y_0, y_1).

Randomness without random tape. *Alice* is not committed to a fixed value of b in the cheating protocol. This is not breaking the protocol, because even in the classical world one could easily construct *Alice*'s strategy so that the attack does not define a fixed bit: she only has to choose the bit at random. In fact, the distribution of probability for the variables in the cheating protocol is exactly the same as in the honest protocol. So, in any reasonable definition of security, one cannot require that *Alice* is committed to a fixed bit defined by the attack.

However, there is still a fundamental difference between the classical situation and the quantum situation. In the classical world, one can look at *Alice*'s random tape and actually determine the bit. So the attack and the random tape together determine the bit. This is why we intuitively think that *Alice* is committed to a fixed bit. This is not true anymore in the quantum case. We cannot think anymore that *Alice* is committed to a fixed bit (determined by the value of the random tape). In a quantum protocol, the outcomes of measurements introduce some randomness which cannot be explained by the use of a random tape. There is no such a thing as a random tape which uniquely determine the bit.

This means that the naïve definition of security associated with classical bit commitment, namely that *Alice* must be committed to a fixed bit, is not valid anymore in the quantum world. This is what we consider a weak quantum attack on classical bit commitment. This is why the security criteria proposed in [2] is that *Alice* should be committed to a random distribution of probability. This notion of security is valid in both the quantum and the classical world.

The fact that in a quantum protocol there is randomness without initial random tape has far reaching consequences (other than simply attacking our naïve notion of security for classical bit commitment). One important ingredient in the proof of NOVY, which reduces the security of their classical bit commitment against $\mathcal{A}lice$ to the existence of one way permutation, is that the randomized strategy used by $\mathcal{A}lice$ is replaced by a deterministic strategy by fixing the value of the random tape. Unfortunately, because randomness can still exist in a quantum protocol even if we fix the initial random tape, this approach does not work. One cannot consider that $\mathcal{A}lice$ performs a fixed strategy. So the proof of Novy does not (at the least not directly) apply to quantum attacks. This is another kind of quantum attacks against classical bit commitment: here it's the proof of security that is attacked, not the protocol directly.

3.2 Two-Prover Bit Commitment Scheme

Our second example is the two-prover BC scheme of BGKW. The assumption used for this protocol is that two parties $\mathcal{A}lice$ and $\mathcal{A}lyson$ who are allowed to exchange information before the beginning of the protocol, cannot communicate during the execution of the protocol. Nevertheless, both of them can talk to $\mathcal{B}ob$. This assumption may be implemented by trapping $\mathcal{A}lice$ and $\mathcal{A}lyson$ in Faraday cages or using relativistic effects keeping them separate of a large enough distance. In an initialization phase, $\mathcal{A}lice$ and $\mathcal{A}lyson$ share information necessary to run the commitment protocol.

Protocol 3.5 ($2P/Init$)

1: $\mathcal{A}lice$ picks $r \in_{\mathrm{R}} \{0,1\}^n$, and shares it as r' with $\mathcal{A}lyson$,

2: $\mathcal{A}lice$ and $\mathcal{A}lyson$ are physically split,

In order to commit they run the following

Protocol 3.6 ($2P/Commit(b)$)

1: $\mathcal{B}ob$ sets $m_0 := 0^n$ and picks $m_1 \in_{\mathrm{R}} \{0,1\}^n$, and announces them to $\mathcal{A}lice$,

2: $\mathcal{A}lice$ sends $z := r \oplus m_b$ to $\mathcal{B}ob$.

The commitment is concealing because for each z there exists a unique pair r_0, r_1 such that $r_0 \oplus m_0 = z = r_1 \oplus m_1$. On the other hand, it is binding because $\mathcal{A}lyson$ does not know the value of m_1.

If unveiling is required they run

Protocol 3.7 ($2P/Unveil(b)$)

1: $\mathcal{A}lice$ discloses b and r to $\mathcal{B}ob$,

2: $\mathcal{A}lyson$ discloses r' to $\mathcal{B}ob$,

3: $\mathcal{B}ob$ checks that $r = r'$ and that $z = r \oplus m_b$,

Since $\mathcal{A}lyson$ does not know m_1, she is restricted to disclosing her r' to have non negligeable probability of satisfying $\mathcal{B}ob$.

Ben-Or, Goldwasser, Kilian and Wigderson [6] have used this protocol to prove NP statements in *perfect zero-knowledge*. This follows from the fact that this bit commitment is unconditionally concealing.

3.2.1 Defeating $2P/Commit$

Let R, R' be a pair of quantum registers in state $(\frac{1}{\sqrt{2}}(|00\rangle + |11\rangle))^n$ shared between $\mathcal{A}lice$ and $\mathcal{A}lyson$ before they are physically separated and let $B := \psi = \alpha|0\rangle + \beta|1\rangle$ be a register containing the particle given to $\mathcal{A}lice$ by $\mathcal{B}ob$.

Protocol 3.8 ($2P/Init$)

1: $\mathcal{A}lice$ and $\mathcal{A}lyson$ choose and share a common string at the quantum level, that is, they share registers (R, R') in state $(\frac{1}{\sqrt{2}}|00\rangle + \frac{1}{\sqrt{2}}|11\rangle)^n$,

2: $\mathcal{A}lice$ and $\mathcal{A}lyson$ are physically split,

Commitment is performed by superposition of the honest protocol:

Protocol 3.9 ($2P/Commit(*)$)

1: $\mathcal{B}ob$ sets $m_0 := 0^n$ and picks $m_1 \in_{\mathrm{R}} \{0,1\}^n$, and announces them to $\mathcal{A}lice$,

2: $\mathcal{A}lice$ computes $z := r \oplus m_b$ at the quantum level, that is, she prepares registers B, R, Z in state

$$\sum_{r \in \{0,1\}^n} \frac{1}{\sqrt{2^n}} (\alpha|0, r, r \oplus m_0\rangle + \beta|1, r, r \oplus m_1\rangle)$$

3: $\mathcal{A}lice$ measures Z to get z and sends z to $\mathcal{B}ob$.

After z is measured the global state of registers B, R, Z, R' is

$$\frac{1}{\sqrt{2}}(\alpha|0, z \oplus m_0, z, z \oplus m_0\rangle + \beta|1, z \oplus m_1, z, z \oplus m_1\rangle).$$

Unveiling is performed by measurements on both sides:

Protocol 3.10 ($2P/Unveil(*)$)

1: $\mathcal{A}lice$ measures $B, R_1, ..., R_n$ to get $b, r_1, ..., r_n$, and discloses b, r to $\mathcal{B}ob$,

2: $\mathcal{A}lyson$ measures $R'_1, ..., R'_n$ to get $r'_1, ..., r'_n$ and discloses r' to $\mathcal{B}ob$,

3: $\mathcal{B}ob$ checks that $r = r'$ and that $z = r \oplus m_b$,

If $\mathcal{A}lice$-$\mathcal{A}lyson$ Unveil We now show that this unveiling is always successful. Remember that after z is measured, the global state of registers B, R, Z, R' is

$$\frac{1}{\sqrt{2}}(\alpha|0, z \oplus m_0, z, z \oplus m_0\rangle + \beta|1, z \oplus m_1, z, z \oplus m_1\rangle)$$

and thus the bit b has the correct distribution of probability (the one associated with the initial state ψ). Also, one may easily check that in both cases $r = z \oplus m_b = r'$. Therefore, $\mathcal{A}lice$-$\mathcal{A}lyson$ pass the test.

If $\mathcal{A}lice$-$\mathcal{A}lyson$ never Unveil Second we observe that if $2P/Unveil(*)$ does not take place, then $\mathcal{A}lice$-$\mathcal{A}lyson$ may recover ψ from their registers B, R, R'. They only have to compute $z \oplus m_b$ at the quantum level and then erase the registers R and R' using a bitwise XOR, and discard these registers. This computation may be done efficiently, but it requires that $\mathcal{A}lice$-$\mathcal{A}lyson$ get back together.

4 Discussion and Conclusions

The first proof provided for the impossibility of bit commitment has completely obliterated the possibility of creating an unconditionally secure bit commitment. However, the attack was only indirectly described. Subsequently, specific attempts to by-pass this general result were proposed[3, 5]. This has shed more light on the nature of the attack which was finally described explicitly in [2]. Our goal here was to provide an analysis of this general attack in the context of a specific example, and to create a wholeness for the different papers published on the subject. Moreover, we have demonstrated that it is impossible to base the security of quantum protocols on unconditionally concealing bit commitment schemes, even if they were proven secure in the classical world. Notice however that it is still possible to use computationally concealing BC protocols such as [10] to get a computationally secure Quantum Oblivious Transfer [23, 14] protocol based on (quantum) one-way functions; a result unlikely to be true in the classical scenario. The big lesson to learn from all this is that quantum information is always more elusive than its classical counterpart: extra care must be taken when reasoning about quantum cryptographic protocols and analyzing them. We also hope that this paper will help to clarify the issue of the impossibility of bit commitment in its full generality.

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